This is what I do:

(restrict representations to compact subgroups)

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Outline

Introduction

Examples

How do you do that?

Applications

And now a word from our sponsor

What are the questions?

Equivariant K-theory

K-theory and representations

Setting

Compact groups K are relatively easy...

Noncompact groups G are relatively hard.

Harish-Chandra *et al.* idea:

$$\text{understand } \pi \in \widehat{G} \rightsquigarrow \text{understand } \pi|_K$$

(nice compact subgroup $K \subset G$).

Get an invariant of a repn $\pi \in \widehat{G}$:

$$m_\pi \colon \widehat{K} \to \mathbb{N}, \qquad m_\pi(\mu) = \text{mult of } \mu \text{ in } \pi|_K.$$

1. What's the support of m_π? (subset of $\widehat{K}$)
2. What's the rate of growth of m_π?
3. What functions on $\widehat{K}$ can be m_π?

Where can you do this?

Get an invariant of a repn $\pi \in \widehat{G}$:

$$m_\pi \colon \widehat{K} \to \mathbb{N}, \qquad m_\pi(\mu) = \text{mult of } \mu \text{ in } \pi|_K.$$

1. What's the support of m_π? (subset of $\widehat{K}$)
2. What's the rate of growth of m_π?
3. What functions on $\widehat{K}$ can be m_π?

I look at $G = G(\mathbb{R})$ real reductive, $K = K(\mathbb{R})$ max cpt.

Questions are just as interesting, and much less understood, for G p-adic, K compact open.

What's an answer look like?

Seek to understand mult of μ in $\pi|_K$ (π irr of G, μ irr of K).

m_π is an $\mathbb{N}$-valued function on an infinite countable set.

One kind of answer:

1. find family $\{S_p \mid p \in P\}$ of functions $S_p \colon \widehat{K} \to \mathbb{Z}$.
2. find nice interpretation of each S_p.
3. find algorithm to write $m_\pi = \sum_{p \in P} a_\pi^p S_p$ (finite sum).
4. find algorithm to compute each S_p.

(3) + (4) computes m_π, (2) gives meaning to answer.

Today $P = \{$ tempered reps of real infl char $\}$, $S_p = m_p$ (branching rule for tempered p).

Examples

1. $G = GL(n, \mathbb{C})$, $K = U(n)$. Typical restriction to K is
$$\pi|_K = \mathrm{Ind}_{U(1)^n}^{U(n)}(\gamma) = \sum_{\mu \in \widehat{U(n)}} m_\mu(\gamma)\gamma \quad (\gamma \in \widehat{U(1)^n}) :$$
$m_\pi(\mu) =$ mult of μ is $m_\mu(\gamma) =$ dim of γ wt space.

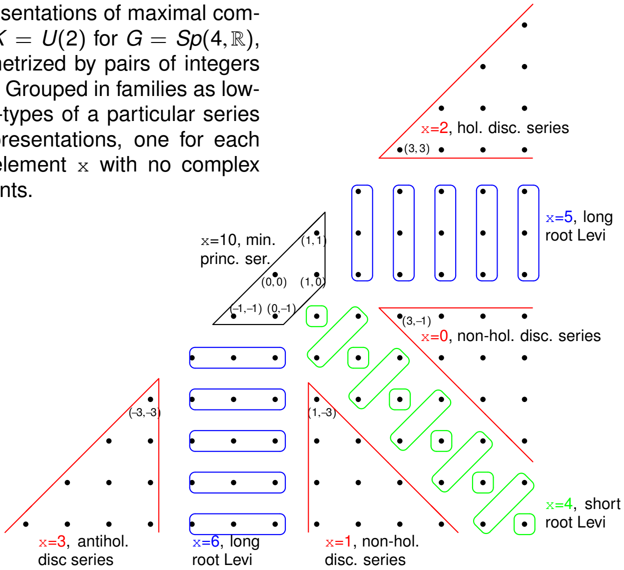
2. $G = GL(n, \mathbb{R})$, $K = O(n)$. Typical restriction to K is
$$\pi|_K = \mathrm{Ind}_{O(1)^n}^{O(n)}(\gamma) = \sum_{\mu \in \widehat{O(n)}} m_\mu(\gamma) :$$
$m_\pi(\mu) =$ mult of μ in π is $m_\mu(\gamma) =$ mult of γ in $\mu|_{O(1)^n}$.

3. G split of type E_8, $K = Spin(16)$. Typical res to K is
$$\pi|_{Spin(16)} = \mathrm{Ind}_M^{Spin(16)}(\gamma) = \sum_{\mu \in \widehat{Spin(16)}} m_\mu(\gamma)\gamma;$$
here $M \subset Spin(16)$ subgp of order 512, central ext of $(\mathbb{Z}/2\mathbb{Z})^8$.

Moral: The hard work of computing m_π takes place inside the world of compact groups.

Lowest K-types for $Sp(4,\mathbb{R})$

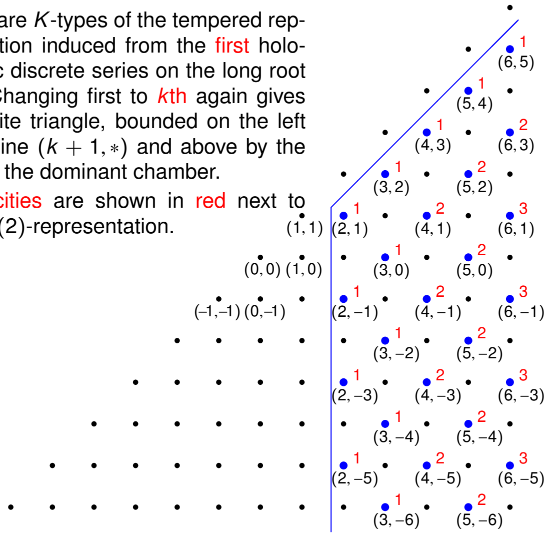
Representations of maximal compact $K = U(2)$ for $G = Sp(4,\mathbb{R})$, parametrized by pairs of integers $a \geq b$. Grouped in families as lowest K-types of a particular series of representations, one for each `KGB` element `x` with no complex descents.



Branching law for ONE tempered rep

In blue are K-types of the tempered representation induced from the first holomorphic discrete series on the long root Levi. Changing first to kth again gives an infinite triangle, bounded on the left by the line $(k+1,*)$ and above by the edge of the dominant chamber.

Multiplicities are shown in red next to each $U(2)$-representation.



How do you do that?

Main ideas are due to Harish-Chandra, Langlands, Schmid, and Knapp-Zuckerman.

Description below makes it sound like they're due to me.

Reason: (given my age) I interpreted the topic as What I did.

Start with preorder on $\widehat{K}$, roughly length of highest weight.

Every rep $\pi \in \widehat{G}$ has one or more lowest K-types μ.

Study all G-reps π of lowest K-type μ.

Find: lowest K-type condition $\Longrightarrow$ Lie alg cohom(π) $\neq 0$.

WHICH Lie alg cohom depends on how singular μ is.

Gives the eight familes in picture of LKT's for $Sp(4, \mathbb{R})$.

More generic $\mu \rightsquigarrow$ better cohom $\rightsquigarrow$ fewer π with LKT μ.

Most generic $\mu \rightsquigarrow$ unique ("discrete series") rep of G with LKT μ.

Four red regions in $\widehat{K}$ in $Sp(4, \mathbb{R})$ picture.

Least generic $\mu \rightsquigarrow$ rk G-param fam ("princ ser") reps with LKT μ.

Black region of five K-types in $Sp(4, \mathbb{R})$ picture.

What do those methods tell you?

Theorem (Langlands) Reductive $G \supset K$ max compact.

1. $\mu \in \widehat{K} \rightsquigarrow H(\mathbb{R}) = TA \subset G$ Cartan subgroup.
2. $T = H \cap K$ compact, $A \overset{\exp}{\simeq} \mathfrak{a}_0$ split vector group.
3. $\mu \in \widehat{K} \rightsquigarrow \lambda \in \widehat{T}$. Precisely: genuine char of $\widetilde{T}_\rho$, M-regular.
4. Put $P = MAN$ (cuspidal) parabolic,

   $\delta \in \widehat{M}$ discrete series with HC param λ.
5. $\nu \in \mathfrak{a}^* \rightsquigarrow$, std rep $I(\lambda, \nu) = \mathrm{Ind}_P^G(\delta \otimes \nu \otimes 1)$
6. $I(\lambda, \nu)$ has μ as a LKT, multiplicity one.
7. $I(\lambda, \nu)$ has unique irr $J(\lambda, \nu)(\mu) \in \widehat{G}$ containing μ.
8. Every $\pi \in \widehat{G}$ of LKT μ is $J(\lambda, \nu)(\mu)$, some $\nu \in \mathfrak{a}^*$.
9. If $\nu = 0$, μ is the unique LKT of $J(\lambda, 0)(\mu)$.

Consequence: reps of LKT μ indexed by cplx vec space $\mathfrak{a}^*$.

Composition series and characters

Prev slide exactly right; this slide needs care with lim disc ser.

Theory of lowest K types start with preorder on $\widehat{K}$.

Langlands classification gives bijection $\widehat{K} \leftrightarrow (T, \lambda)$.

$\rightsquigarrow$ preorder on pairs (T, λ), inherited by params (TA, λ, ν).

Each std has finite comp series $I(\lambda, \nu) = \sum_{\lambda',\nu'} m_{\lambda,\nu}^{\lambda',\nu'} J(\lambda', \nu')$.
Nonnegative integer coeffs $m_{\lambda,\nu}^{\lambda',\nu'}$, $(\lambda', \nu') \geq (\lambda, \nu)$.
Equality in preorder only if $(\lambda', \nu') = (\lambda, \nu)$, and then $m_{\lambda,\nu}^{\lambda',\nu'} = 1$.

Each irr has finite char formula $J(\lambda, \nu) = \sum_{\lambda',\nu'} M_{\lambda,\nu}^{\lambda',\nu'} I(\lambda', \nu')$.
Integer coeffs $M_{\lambda,\nu}^{\lambda',\nu'}$, $(\lambda', \nu') \geq (\lambda, \nu)$.
Equality in preorder only if $(\lambda', \nu') = (\lambda, \nu)$, and then $M_{\lambda,\nu}^{\lambda',\nu'} = 1$.

Matrices $m_{\lambda,\nu}^{\lambda',\nu'}$ and $M_{\lambda,\nu}^{\lambda',\nu'}$ are integer, upper triang, 1s on diag.
They are inverse to each other.

$M_{\lambda,\nu}^{\lambda',\nu'}$ is computed by Kazhdan-Lusztig theory.

Restriction to K

Recall that plan to compute $\pi|_K$ was to write multiplicity function m_π as finite sum of nice functions M_p.

We've done that:

$$\pi = \sum_{\lambda',\nu'} M_\pi^{\lambda',\nu'} I(\lambda',\nu')$$

$$\pi|_K = \sum_{\lambda',\nu'} M_\pi^{\lambda',\nu'} I(\lambda',\nu')|_K = \sum_{\lambda',\nu'} M_\pi^{\lambda',\nu'} I(\lambda',0)|_K$$

$$\pi|_K = \sum_{\lambda'} a_\pi^{\lambda'} I(\lambda',0)|_K.$$

Here $a_\pi^{\lambda'} = \sum_{\nu'} M_\pi^{\lambda',\nu'}$, and $\{I(\lambda',0)\} = \{$ tempered, real infl char $\}$.

Remains to compute $I(\lambda',0)|_K$. A great story, not told today.

What good is this?

Original problem for inf-diml reps: which $\pi \in \widehat{G}(\mathbb{R})$ are unitary?

Knapp and Zuckerman: which π admit invt Hermitian form $\langle , \rangle_\pi$.

If form exists, and K rep μ has multiplicity $m_\pi(\mu)$, then "form restricted to μ gives signature $(p_\pi(\mu), q_\pi(\mu))$.

Find in this way two new functions p_π, q_π from $\widehat{K}$ to $\mathbb{N}$.

Like m_π, $p_\pi,\ q_\pi =$ finite integer combs of $I(\lambda', 0)|_K$.

Compute signature $\leftrightsquigarrow$ compute finitely many integers.

This sounds like a job for a computer.

[Adams/van Leeuwen/Trapa/DV], Astérisque **417**: it is.

How do you compute these things really?

A lot of this mathematics is twentieth century.

Around 2000, Jeffrey Adams set out to make software implementing all these algorithms.

He succeeded, mostly because he managed to interest Fokko du Cloux and Marc van Leeuwen.

Software is at `www.liegroups.org/`.

enter any real reductive `G`, any parameter `p`.

Then can type

```
composition_series(p)
character_formula(p)
print_branch_irr(p,[height])
is_unitary(p)
```

…and much more.

Plan for today

Remaining slides (not presented): work with Jeff Adams to compute associated varieties of G representations.

Work with real reductive Lie group $G(\mathbb{R})$.

Describe (old) associated cycle $\mathcal{AC}(\pi)$ for irr rep $\pi \in \widehat{G(\mathbb{R})}$: geometric shorthand for approximating restriction to $K(\mathbb{R})$ of π.

Describe algorithm with Adams to compute $\mathcal{AC}(\pi)$.

A *real* algorithm is one that's been implemented on a computer. This one has been, by Adams in the `atlas` software.

Assumptions

$G(\mathbb{C}) = G =$ cplx conn reductive alg gp.

$G(\mathbb{R}) =$ group of real points for a real form.

Could allow fin cover of open subgp of $G(\mathbb{R})$, so allow nonlinear.

$K(\mathbb{R}) \subset G(\mathbb{R})$ max cpt subgp; $K(\mathbb{R}) = G(\mathbb{R})^\theta$.

$\theta =$ alg inv of G; $K = G^\theta$ possibly disconn reductive.

Harish-Chandra idea:

∞-diml reps of $G(\mathbb{R})$ $\leftrightsquigarrow$ alg gp $K \curvearrowright$ cplx Lie alg $\mathfrak{g}$

$(\mathfrak{g}, K)$-module is vector space V with

1. repn π_K of algebraic group K: $V = \sum_{\mu \in \widehat{K}} m_V(\mu)\mu$
2. repn $\pi_\mathfrak{g}$ of cplx Lie algebra $\mathfrak{g}$
3. $d\pi_K = \pi_\mathfrak{g}|_\mathfrak{k}, \qquad \pi_K(k)\pi_\mathfrak{g}(X)\pi_K(k^{-1}) = \pi_\mathfrak{g}(\mathrm{Ad}(k)X)$.

In module notation, cond (3) reads $k \cdot (X \cdot v) = (\mathrm{Ad}(k)X) \cdot (k \cdot v)$.

Geometrizing representations

$G(\mathbb{R})$ real reductive, $K(\mathbb{R})$ max cpt, $\mathfrak{g}(\mathbb{R})$ Lie alg

$\mathcal{N}^* =$ cone of nilpotent elements in $\mathfrak{g}^*$.

$\mathcal{N}^*_{\mathbb{R}} = \mathcal{N}^* \cap i\mathfrak{g}(\mathbb{R})^*$, finite # $G(\mathbb{R})$ orbits.

$\mathcal{N}^*_{\theta} = \mathcal{N}^* \cap (\mathfrak{g}/\mathfrak{k})^*$, finite # K orbits.

Goal 1: Attach orbits to representations in theory.

Goal 2: Compute them in practice.

"In theory there is no difference between theory and practice. In practice there is." Jan L. A. van de Snepscheut (or not).

$(\pi, \mathcal{H}_\pi)$ irr rep of $G(\mathbb{R})$	$\mathcal{H}_\pi^K$ irr $(\mathfrak{g}, K)$-module
$\downarrow$ Howe wavefront	$\downarrow$ assoc var of gr
$\mathrm{WF}(\pi) = G(\mathbb{R})$ orbs on $\mathcal{N}^*_{\mathbb{R}}$	$\mathcal{AC}(\pi) = K$ orbits on $\mathcal{N}^*_\theta$

Columns related by HC, Kostant-Rallis, Sekiguchi, Schmid-Vilonen.

So Goal 1 is completed. Turn to Goal 2...

Associated varieties

$\mathcal{F}(\mathfrak{g}, K)$ = finite length $(\mathfrak{g}, K)$-modules...

noncommutative world we care about.

$\mathcal{C}(\mathfrak{g}, K) =$ f.g. $(S(\mathfrak{g}/\mathfrak{k}), K)$-modules, support $\subset \mathcal{N}_\theta^*$...

commutative world where geometry can help.

$$\mathcal{F}(\mathfrak{g}, K) \overset{\text{gr}}{\rightsquigarrow} \mathcal{C}(\mathfrak{g}, K)$$

gr not quite a functor (choice of good filts), but

Prop. gr induces surjection of Grothendieck groups

$$K\mathcal{F}(\mathfrak{g}, K) \xrightarrow{\text{gr}} K\mathcal{C}(\mathfrak{g}, K);$$

image records restriction to K of HC module.

So restrictions to K of HC modules sit in equivariant coherent sheaves on nilp cone in $(\mathfrak{g}/\mathfrak{k})^*$

$$K\mathcal{C}(\mathfrak{g}, K) =_{\text{def}} K^K(\mathcal{N}_\theta^*),$$

equivariant K-theory of the K-nilpotent cone.

Goal 2: compute $K^K(\mathcal{N}_\theta^*)$ and the map **Prop.**

Equivariant K-theory

Setting: (complex) algebraic group K acts on (complex) algebraic variety X.

Originally K-theory was about vector bundles, but for us coherent sheaves are more useful.

$\mathrm{Coh}^K(X) =$ abelian categ of coh sheaves on X with K action.

$K^K(X) =_{\mathrm{def}}$ Grothendieck group of $\mathrm{Coh}^K(X)$.

Example: $\mathrm{Coh}^K(\mathrm{pt}) = \mathrm{Rep}(K)$ (fin-diml reps of K).

$K^K(\mathrm{pt}) = R(K) =$ rep ring of K; free $\mathbb{Z}$-module, basis $\widehat{K}$.

Example: $X = K/H$; $\mathrm{Coh}^K(K/H) \simeq \mathrm{Rep}(H)$

$E \in \mathrm{Rep}(H) \rightsquigarrow \mathcal{E} =_{\mathrm{def}} K \times_H E$ eqvt vector bdle on K/H

$K^K(K/H) = R(H)$.

Example: $X = V$ vector space.

$E \in \mathrm{Rep}(K) \rightsquigarrow$ proj module $\mathcal{O}_V(E) =_{\mathrm{def}} \mathcal{O}_V \otimes E \in \mathrm{Coh}^K(X)$

proj resolutions $\Longrightarrow$ $K^K(V) \simeq R(K)$, basis $\{\mathcal{O}_V(\tau)\}$.

Doing nothing carefully

Suppose $K \curvearrowright X$ with finitely many orbits:

$X = Y_1 \cup \cdots \cup Y_r, \qquad Y_i = K \cdot y_i \simeq K/K^{y_i}$.

Orbits partially ordered by $Y_i \geq Y_j$ if $Y_j \subset \overline{Y_i}$.

$(\tau, E) \in \widehat{K^{y_i}} \rightsquigarrow \mathcal{E}(\tau) \in \mathrm{Coh}^K(Y_i)$.

Choose (always possible) K-eqvt coherent extension

$$\widetilde{\mathcal{E}}(\tau) \in \mathrm{Coh}^K(\overline{Y_i}) \rightsquigarrow [\widetilde{\mathcal{E}}] \in K^K(\overline{Y_i}).$$

Class $[\widetilde{\mathcal{E}}]$ on $\overline{Y}_i$ unique modulo $K^K(\partial Y_i)$.

Set of all $[\widetilde{\mathcal{E}}(\tau)]$ (as Y_i and τ vary) is basis of $K^K(X)$.

Suppose $M \in \mathrm{Coh}^K(X)$; write class of M in this basis

$$[M] = \sum_{i=1}^{r} \sum_{\tau \in \widehat{K^{y_i}}} n_\tau(M)[\widetilde{\mathcal{E}}(\tau)].$$

Maxl orbits in $\mathrm{Supp}(M)$ = maxl Y_i with some $n_\tau(M) \neq 0$.

Coeffs $n_\tau(M)$ on maxl Y_i ind of choices of exts $\widetilde{\mathcal{E}}(\tau)$.

Our story so far

We have found

1. homomorphism

$$\text{virt } G(\mathbb{R}) \text{ reps } \; K\mathcal{F}(\mathfrak{g}, K) \xrightarrow{\text{gr}} K^K(\mathcal{N}_\theta^*) \; \text{ eqvt } K\text{-theory}$$

2. geometric basis $\left\{[\widetilde{\mathcal{E}(\tau)}]\right\}$ for $K^K(\mathcal{N}_\theta^*)$, indexed by irr reps of isotropy gps
3. expression of $[\text{gr}(\pi)]$ in geom basis $\rightsquigarrow \mathcal{AC}(\pi)$.

Problem is expressing ourselves. . .

Teaser for the next section: Kazhdan and Lusztig taught us how to express π using std reps $I(\gamma)$:

$$[\pi] = \sum_\gamma m_\gamma(\pi)[I(\gamma)], \qquad m_\gamma(\pi) \in \mathbb{Z}.$$

$\{[\text{gr } I(\gamma)]\}$ is another basis of $K^K(\mathcal{N}_\theta^*)$.

Last goal is compute change of basis matrix.

The last goal

Studying cone $\mathcal{N}_\theta^* =$ nilp lin functionals on $\mathfrak{g}/\mathfrak{k}$.

Found (for free) basis $\left\{[\widetilde{\mathcal{E}(\tau)}]\right\}$ for $K^K(\mathcal{N}_\theta^*)$, indexed by orbit K/K^i and irr rep τ of K^i.

Found (by rep theory) second basis $\{[\mathrm{gr}\, I(\gamma)]\}$, indexed by (parameters for) std reps of $G(\mathbb{R})$.

To compute associated cycles, enough to write

$$[\mathrm{gr}\, I(\gamma)] = \sum_{\text{orbits}} \sum_{\substack{\tau \text{ irr for} \\ \text{isotropy}}} N_\tau(\gamma)[\widetilde{\mathcal{E}}(\tau)].$$

Equivalent to compute inverse matrix

$$[\widetilde{\mathcal{E}}(\tau)] = \sum_\gamma n_\gamma(\tau)[\mathrm{gr}\, I(\gamma)].$$

Need to relate geom of nilp cone to geom std reps: parabolic subgroups. Use Springer resolution.

Introducing Springer

$\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{s}$ Cartan decomp, $\mathcal{N}_\theta^* \simeq \mathcal{N}_\theta =_{\text{def}} \mathcal{N} \cap \mathfrak{s}$ nilp cone in $\mathfrak{s}$.

Kostant-Rallis, Jacobson-Morozov: nilp $X \in \mathfrak{s} \rightsquigarrow Y \in \mathfrak{s},\ H \in \mathfrak{k}$

$$[H, X] = 2X, \quad [H, Y] = -2Y, \quad [X, Y] = H,$$

$$\mathfrak{g}[k] = \mathfrak{k}[k] \oplus \mathfrak{s}[k] \qquad (\text{ad}(H) \text{ eigenspace}).$$

$\rightsquigarrow \mathfrak{g}[\geq 0] =_{\text{def}} \mathfrak{q} = \mathfrak{l} + \mathfrak{u}$ θ-stable parabolic.

Theorem (Kostant-Rallis) Write $\mathcal{O} = K \cdot X \subset \mathcal{N}_\theta$.

1. $\mu \colon \mathcal{O}_Q =_{\text{def}} K \times_{Q \cap K} \mathfrak{s}[\geq 2] \to \overline{\mathcal{O}}$, $(k, Z) \mapsto \text{Ad}(k)Z$ is proper birational map onto $\overline{\mathcal{O}}$.
2. $K^X = (Q \cap K)^X = (L \cap K)^X (U \cap K)^X$ is a Levi decomp; so $\widehat{K^X} = [(L \cap K)^X]\widehat{\ }$.

So have resolution of singularities of $\overline{\mathcal{O}}$:

$$\begin{array}{ccc} & K \times_{Q \cap K} \mathfrak{s}[\geq 2] & \\ \text{vec bdle} \swarrow & & \searrow \mu \\ K/Q \cap K & & \overline{\mathcal{O}} \end{array}$$

Use it (*i.e.*, copy McGovern, Achar) to calculate equivariant K-theory. . .

Using Springer to calculate K-theory

$X \in \mathcal{N}_\theta$ represents $\mathcal{O} = K \cdot X$.

$\mu \colon \mathcal{O}_Q =_{\text{def}} K \times_{Q \cap K} \mathfrak{s}[\geq 2] \to \overline{\mathcal{O}}$ Springer resolution.

Theorem Recall $\widehat{K^X} = [(L \cap K)^X]^\frown$.

1. $K^K(\mathcal{O}_Q)$ has basis of eqvt vec bdles:
$$(\sigma, F) \in \text{Rep}(L \cap K) \rightsquigarrow \mathcal{F}(\sigma).$$
2. Get extension of $\mathcal{E}(\sigma|_{(L \cap K)^X})$ on $\mathcal{O}$
$$[\overline{\mathcal{F}}(\sigma)] =_{\text{def}} \sum_i (-1)^i [R^i \mu_*(\mathcal{F}(\sigma))] \in K^K(\overline{\mathcal{O}}).$$
3. Compute (very easily) $[\overline{\mathcal{F}}(\sigma)] = \sum_\gamma n_\gamma(\sigma)[\text{gr}\, I(\gamma)]$.
4. Each irr $\tau \in [(L \cap K)^X]^\frown$ extends to (virtual) rep $\sigma(\tau)$ of $L \cap K$; can choose $\overline{\mathcal{F}(\sigma(\tau))}$ as extension of $\mathcal{E}(\tau)$.

Now we're done

Recall $X \in \mathcal{N}_\theta \rightsquigarrow \mathcal{O} = K \cdot X$; $\tau \in [(L \cap K)^X]\widehat{}$.

Now we know formulas

$$[\widetilde{\mathcal{E}}(\tau)] = [\overline{\mathcal{F}(\sigma(\tau))}] = \sum_\gamma n_\gamma(\tau)[\mathrm{gr}\, I(\gamma)].$$

Here's why this does what we want:

1. inverting matrix $n_\gamma(\tau) \rightsquigarrow$ matrix $N_\tau(\gamma)$ writing $[\widetilde{\mathcal{E}}(\tau)]$ in terms of $[\mathrm{gr}\, I(\gamma)]$.
2. multiplying $N_\tau(\gamma)$ by Kazhdan-Lusztig matrix $m_\gamma(\pi)$ $\rightsquigarrow$ matrix $n_\tau(\pi)$ writing $[\mathrm{gr}\, \pi]$ in terms of $[\widetilde{\mathcal{E}}(\tau)]$.
3. Nonzero entries $n_\tau(\pi) \rightsquigarrow \mathcal{AC}(\pi)$.

Side benefit: algorithm (for $G(\mathbb{R})$ cplx) also computes bijection (conj by Lusztig, estab by Bezrukavnikov)

$$(\text{dom wts}) \leftrightarrow (\text{pairs } (\tau, \mathcal{O}))$$