2017 SPUR Conference

Friday, August 4

Room 2-449

SCHEDULE

9:00 am	Conference Opening, by SPUR Faculty Advisors Prof. David Jerison and Prof. Ankur Moitra, and SPUR+ Coordinator Dr. Cris Negron
9:10 am	Max Murin, *Computing Parameters for Generalized Hecke Algebras in Type B* (mentor Seth Shelley-Abrahamson)
9:40 am	Panagiotis Dimakis, *On a Connection between Affine Springer Fibers and L-U Decomposable Matrices* (mentor Guangyi Yue)
10:10 am	Jianqiao Xia, *Topology of Two-Row Type Springer Fibers* (mentor Gus Lonergan)
10:40 am	**Break** (10:40 – 10:50)
10:50 am	Hyo Won Kim and Christopher Maldonado, *On Graphs and the Gotsman-Linial Conjecture for* $d = 2$ (mentor Jake Lee Wellens)
11:30 am	Max Vargas and Kimberly Villalobos, *A Random Walk Formulation of Learning in Restricted Boltzmann Machines* (mentor Mason Biamonte)
12:10 pm	**Lunch break** (12:10 – 1:00)
1:00 pm	Xianglong Ni, *The Bracket in the Bar Spectral Sequence for a Finite-Fold Loop Space* (mentor Hood Chatham)
1:30 pm	Juan Carlos Ortiz, *The Diagonal Cohomology Class of Vertical Bundles* (mentor Jackson Hance)
2:00 pm	Alonso Espinosa Dominguez, *On Kakeya-Type Problems for Hyperplanes in* $\mathbb{R}^d$ (mentor Paxton Turner)
2:30 pm	**Break** (2:30 – 2:40)
2:40 pm	Brian Chen, *The Suboptimality of Asymmetric Recursive Reconstruction Algorithms* (mentor Ashwin Narayan)
3:10 pm	Justin Lim, *Building Forests in Maker-Breaker Games: Upper and Lower Bounds* (mentor Frederic Koehler)
3:40 pm	Luke Sciarappa, *Model Categories in Equivariant Rational Homotopy Theory* (mentor Robert Burklund)
4:10 pm	Conference Closing

The Suboptimality of Asymmetric Recursive Reconstruction Algorithms

Brian Chen
Mentor: Ashwin Narayan
Project suggested by Elchanan Mossel

We examine a variant of tree reconstruction proposed by Mossel. Given a rooted tree T with d leaves, all at the same level, we consider trees T_n consisting of n levels of copies of T, and randomly label their vertices with bits as follows. The root is labeled with a random bit, and then for each child, it is labeled with a bit that differs from its parent with probability ε. We analyze algorithms that, given guesses for the d labels of leaves of a copy of T, output a guess for the root of T, which can be recursively applied starting from labels of the leaves of each T_n to obtain a guess for the label of the root of T_n. An algorithm achieves recursive reconstruction if its probability of correctly guessing the label of the root is bounded away from $1/2$. In this paper, we show that prior analysis of recursive reconstruction algorithms do not apply to asymmetric algorithms that have different probabilities of reconstructing a 0 versus a 1. In particular, asymmetric majority algorithms that output the most common label among the children but break ties unevenly fail to achieve recursive reconstruction for a range of ε where symmetric majority algorithms succeed. We also discuss some empirical evidence and theoretical difficulties in studying generalizations of this model.

On a Connection between Affine Springer Fibers and L-U Decomposable Matrices

Panagiotis Dimakis
Mentor: Guangyi Yue
Project suggested by Roman Bezrukavnikov

In this paper we study the spaces X defined as $\mathbb{O} \cap B_-B_+$ where $\mathbb{O}$ is a regular semi-simple orbit of a semi-simple group G over $\mathbb{C}$ and B_-, B_+ are opposite Borel subgroups in G in the special case where $G = SL_n$. We describe a conjectural correspondence between affine Springer fibers over SL_n and the above spaces and verify it when $G = SL_2$. Finally we give a conjecture about the shape of X when $G = SL_3$ based on the above correspondence.

On Kakeya-Type Problems for Hyperplanes in $\mathbb{R}^d$

Alonso Espinosa Domínguez
Mentor: Paxton Turner
Project suggested by Larry Guth

It is well known that in $\mathbb{R}^2$, there exist so-called Kakeya sets, which are compact measure zero sets containing a unit line segment in every direction. It is also well known that for $d \geq 3$, a compact set with a unit $d-1$ cube in every direction cannot have measure zero. For $E \subset \mathbb{R}^d$, $d \geq 3$ and $A \in \mathbb{R}$, we first consider bounds on the measure of sets $\Gamma_A(E)$ of directions on $\mathbb{S}^{d-1}$ for which there is a hyperplane slice of E with $d-1$ dimensional measure larger than A. In particular, we study certain families of ellipsoids in $\mathbb{R}^3$ to compare the size of their $\Gamma_A(E)$ sets to previously known bounds. Then, in an attempt to develop tools to analyze $\Gamma_A(E)$, we use a maximal operator related to the Kakeya maximal function and derive estimates that elucidate some properties of Kakeya-like sets containing a unit $d-1$ cube normal to every direction.

On Graphs and the Gotsman-Linial Conjecture for $d = 2$

Hyo Won Kim and Christopher Maldonado
Mentor Jake Lee Wellens
Project suggested by Jake Lee Wellens

Given a polynomial $p(x) : \{-1,1\}^n \to \mathbb{R}^\times$, the associated polynomial threshold function (PTF) is a boolean function $f : \{-1,1\}^n \to \{-1,1\}$ defined by $f(x) = \text{sgn}\,(p(x))$. A conjecture of Gotsman and Linial posits that among all degree-d PTFs, the one with the largest influence corresponds to the symmetric degree-d polynomial which alternates sign at the $d+1$ values of $\sum_{i=1}^n x_i$ closest to 0, having influence $\Theta(d\sqrt{n})$. We give a counterexample when $d = 2$, $n = 5$, thereby disproving the conjecture as originally stated. However, as a theorem of Kane shows, for constant d, and any degree-d PTF f on n variables, $\mathbf{I}[f] = O(\sqrt{n} \cdot \text{poly}\,\log(n))$, so at least the conjectured bound $O(d\sqrt{n})$ is not too far off for small d. We examine the case $d = 2$, i.e. when $f(x) = \text{sgn}\,(x^T Ax + b^T x + c)$, and using only elementary methods, we remove the poly $\log(n)$ from Kane's bound in a variety of special cases, based on graph properties of the matrix A, interpreted as a weighted adjacency matrix.

Building Forests in Maker-Breaker Games: Upper and Lower Bounds

Justin Lim
Mentor: Frederic Koehler
Project suggested by Asaf Ferber

We give explicit "linear-time" strategies for building spanning forests in $1:1$ Maker-Breaker games and matching lower bounds in some interesting cases, e.g. for a forest of k-cycles.

Computing Parameters for Generalized Hecke Algebras in Type B

Max Murin
Mentor: Seth Shelley-Abrahamson
Project suggested by Seth Shelley-Abrahamson

The irreducible representations of full support in the rational Cherednik category $\mathcal{O}_c(W)$ attached to a Coxeter group W are in bijection with the irreducible representations of an associated Iwahori-Hecke algebra. Recent work of Losev and Shelley-Abrahamson has shown that the irreducible representations in $\mathcal{O}_c(W)$ of arbitrary given support are similarly governed by certain generalized Hecke algebras. In this paper we compute the parameters for these generalized Hecke algebras in the remaining previously unknown cases, corresponding to the parabolic subgroup $B_n \times S_k$ in B_{n+k} for $k \geq 2$ and $n \geq 0$.

The Bracket in the Bar Spectral Sequence for a Finite-Fold Loop Space

Xianglong Ni
Mentor: Hood Chatham
Project suggested by Haynes Miller

When X is an associative H-space, the bar spectral sequence computes the homology of the delooping, $H_*(BX)$. If X is an n-fold loop space for $n \geq 2$ this is a spectral sequence of Hopf algebras. Using machinery by Sugawara and Clark, we show that the spectral sequence filtration respects the Browder bracket structure on $H_*(BX)$, and so it is moreover a spectral sequence of Poisson algebras. Through the bracket on the spectral sequence, we establish a connection between the degree $n-1$ bracket on $H_*(X)$ and the degree $n-2$ bracket on $H_*(BX)$. This generalizes a result of Browder and puts it in a computation-ready context.

The Diagonal Cohomology Class of Vertical Bundles

Juan Carlos Ortiz
Mentor: Jackson Hance
Project suggested by Jackson Hance

Given a manifold M, Milnor and Stasheff studied the *diagonal cohomology class* $u'' \in H^m(M \times M; \mathbb{Z}/2)$ that describes the orientation of the tangent bundle, and is related to its Stiefel-Whitney Classes. We generalize this concept to fiber bundles $M \to E \to N$ where the fiber is a manifold and the base is a Poincaré space, study the naturality of the construction, give further characterizations of the class, and compute it for certain examples.

Model Categories in Equivariant Rational Homotopy Theory

Luke Sciarappa
Mentor: Robert Burklund
Project suggested by Robert Burklund

Model categories are a useful formalization of homotopy theory, and the notion of Quillen equivalence between them expresses what it means for two homotopy theories to be equivalent. Bousfield and Gugenheim showed that the model categories of simplicial sets and of commutative differential graded algebras over $\mathbb{Q}$ are close to Quillen equivalent, in that there is a Quillen adjunction between them such that the induced adjunction on homotopy categories restricts to an equivalence on simply-connected rational objects of finite rational type. We extend this to the case of an action of a discrete group, showing that for any small category C the induced Quillen adjunction between functors from C to simplicial sets and functors from C to commutative differential graded algebras also restricts to an equivalence on appropriately well-behaved objects. We also explore equivariant rational homotopy theory from this perspective, for the action of a finite group.

A Random Walk Formulation of Learning in Restricted Boltzmann Machines

Max Vargas and Kimberly Villalobos
Mentor: Mason Biamonte
Project suggested by Mason Biamonte

Mehta and Schwab (2014) have conjectured that there exists a mapping between learning in restricted Boltzmann machines and the renormalization group which arises in the study of phase transitions in Ising-type spin models and in exorcising infinities that emanate in quantum field theory. Rigorous approaches based on the theory of random walks have been developed by Brydges, Frölich, and Spencer (1982) as well as Aizenmann (1985) in order to explain phenomena behind Ising-type models and the renormalization group, respectively. Here we use similar methods to take a step towards completing the formal connection between the renormalization group and learning on a RBM by constructing a random walk representation of correlation functions arising from minimization of the Kullback-Leibler divergence. This random walk representation casts deep learning in the framework of quantum field theory and allows for analysis and generalizations of learning algorithms that arise naturally in a field theory setting.

Topology of Two-row Type Springer Fibers

Jianqiao Xia
Mentor: Gus Lonergan
Project Suggested by Roman Bezrukavnikov

It is known that irreducible components of a two-row type Springer fiber are iterated $\mathbb{CP}^1$ bundles. In this paper, we prove that their pair-wise intersections are isomorphic to some irreducible components of other two-row type Springer fibers. In particular, the intersections are iterated $\mathbb{CP}^1$ bundles, as conjectured by Fung. We relate the irreducible components with lattice paths and give an combinatorial algorithm to determine their intersections. For any two-row type Springer fiber, we prove that its singular locus is equidimensional. We find a bound on the number of components of this singular locus and investigate its topology. We make some speculations here, including that the singular locus is a union of Springer fibers of two-row type.