Madeleine de Belloy

Computing the Mosaic Number of Reduced Projections of Knots and Links

under the direction of Mary Stelow

Abstract

In 2008, Lomonaco and Kauffman introduced the concept of knot mosaics, which are diagrams representing knots on $n \times n$ grids. They defined an invariant, called the knot mosaic number, which is the smallest n such that the knot can be represented on an $n \times n$ mosaic. We provide elementary ways of computing the knot mosaic number of reduced projections of certain knots and links without completing an exhaustive search of small mosaic boards. We also answer a question asked by Lee and others by proving that the 8_3 knot does not fit in reduced form on a 6×6 board.

Ruben Mason Carpenter

On Chaotic Pilot Wave Dynamics

under the direction of Nicholas Zhixian Liu

Abstract

A millimetric droplet can bounce indefinitely on a vibrating fluid bath and will self propel along the surface influenced by its underlying wave field, provided the vibration frequency is below the Faraday threshold. Every impact generates a new wave which adds towards the piloting wave field and simultaneously the previously generated wave kernels decay over time. Hence the dynamical system accumulates a total memory which correlates to the strength of the exerted wave force. We focus on the case where the droplet is also subject to a harmonic potential and its motion is constrained to a line. By analyzing the limit cycles in cases of high memory, we observe wave dominated trajectories resembling a growth-relaxation process that has been reported in previous literature studies in the absence of a spring force. Heuristically, as we progress towards chaotic behavior, we extend this type of behavior to unsteady walking dynamics where the quantized trajectory resembles a random walk. We numerically implement a variational method for approaching periodic orbits in a general flow, which we use for determining the nature of unstable periodic orbits arising in a random walk. We thus provide an alternate understanding of this behavior as transitioning between the neighborhoods of these unstable periodic orbits. Finally, we quantify the stability of these by calculating their Floquet multipliers, and we relate their magnitude to the emerging multimodal statistics in of the stationary points.

Parth Chavan

On enumeration of $\Gamma^{a,b,c}$-ideals

under the direction of Jeffery Yu

Abstract

Given a numerical semigroup, the generating function for number of ideals of each size has many known applications in algebraic geometry. Existing methods in literature provide the ideal generating function for the two generator semigroup $\Gamma^{a,b}$. We provide an enumeration and the generating function of the number of $\Gamma^{a,b,c}$-ideals when $b + c$ is a multiple of a. Using this, we explicitly find the ideal generating function for the numerical semigroup $\Gamma^{3,n+2,2n+1}$.

Sarth Chavan

On discriminants of minimal polynomials of the Ramanujan t_n class invariants

under the direction of Alan Peng

Abstract

We study the discriminants of the minimal polynomials $\mathcal{P}_n$ of the Ramanujan t_n class invariants, which are defined for positive $n \equiv 11 \pmod{24}$, and we denote them by $\Delta(\mathcal{P}_n)$. The historical precedent for doing so comes from Gross-Zagier, which is known for computing the prime factorizations of certain resultants and discriminants of the Hilbert class polynomials, which are denoted by H_n. We show that $\Delta(\mathcal{P}_n)$ divides $\Delta(H_n)$ with the quotient being a perfect square. As a consequence, we explicitly determine the sign of $\Delta(\mathcal{P}_n)$ based on the class group structure of the order of discriminant $-n$. Moreover, we also show that the discriminant of the number field $\mathbb{Q}(\mathfrak{j}_n)$ divides $\Delta(\mathcal{P}_n)$, with $\mathfrak{j}_n = \mathbb{Q}\left(j\left(\left(-1+\sqrt{-n}\right)/2\right)\right)$, where j denotes the j-invariant.

Matthew Chen

Mod p Homology of Unordered Configuration Spaces

under the direction of Adela Zhang

Abstract

For topological spaces, computing their homology groups is an algebraic tool for characterizing the complexity of the space—having larger homology groups indicates a more complicated space. Moreover, the homology of a space is a homotopy invariant, allowing one to algebraically determine if two spaces are homotopy equivalent. In this paper, we compute the mod p homology groups of the unordered configuration spaces $B_k(M)$ for various framed manifolds M using a spectral sequence by Knudsen. For genus $g \geq 1$ surfaces Σ_g with $k \leq p$ and $p \geq 5$, we prove that the dimensions of the mod p homology groups of $B_k(\Sigma_g)$ agree with the dimensions of its rational homology group computed by Drummond-Cole and Knudsen. In addition, we compute the mod 2 homology of $B_2(\mathbb{RP}^{2n+1})$ for any odd dimensional real projective space.

Iliyas Noman

On Small Spherical 2-Distance Sets in n-Dimensional Euclidean Space

under the direction of Yuan Yao

Abstract

A set of points S in n-dimensional Euclidean space $\mathbb{R}^n$ is called a *2-distance set* if the set of pairwise distances between the points has cardinality two. The 2-distance set is called *spherical*, if its points lie on the unit sphere in $\mathbb{R}^n$. In $\mathbb{R}^n$ there is a finite number of 2-distance sets with $n+2$ points and an infinite number of 2-distance sets on $n+1$ points and fewer. We characterize the spherical 2-distance sets with $n+2$ points in $\mathbb{R}^n$.

Apoorva Panidapu

Sparse Symmetrizers of Diagonalizable Real-λ Matrices

under the direction of Mo Chen

Abstract

Every diagonalizable $m \times m$ matrix A with real eigenvalues can be symmetrized by some change of basis S; that is, there exists an invertible S such that $H = SAS^{-1}$ is symmetric. In fact, there are infinitely many such S, but we want to specifically find sparse S, such as a diagonal or tridiagonal matrix. To this end, we formulate and code a convex optimization problem that enforces matrix sparsity by minimizing the L^1 norm. We successfully recover sparse outputs for diagonal and diagonally dominant matrices A, but fail to find sparse S for tridiagonal and sparse random matrices.

Rich Wang

Level Spacing for Resonances of Open Quantum Maps

under the direction of Zhenhao Li

Abstract

Open quantum maps provide a simple finite-dimensional model for open quantum systems. The quantum open baker's map is an example of one such map, and examining the behavior of its eigenvalues gives us an idea about how the frequency and decay of waves in open quantum systems behave. We are especially interested in bounds of the magnitudes of the eigenvalues and their distribution in the complex plane. Bounds on the magnitude for the eigenvalue tell us roughly how quickly waves decay. In this paper, we extend a previous result on spectral gaps to a wider family of maps. We also numerically explore the level spacing distributions of systems with the maximal spectral gap.

Xingyan(Summer) Zhou

Products of Values of Polynomials in Finite Fields

under the direction of Rachana Madhukara and Alan Peng

Abstract

In this paper we study products of quadratic and cubic polynomials in finite fields. We expand on the result of Sun in 2019 which determined $\prod_{1\le i<j\le \frac{p-1}{2}}(i^2+j^2)$ for prime $p\equiv 1 \pmod 4$, and generalize it to arbitrary finite fields of odd order. Then, we look at product of polynomials in the form of the sum of two cubics, and determine completely the product $\prod_{\substack{1\le i<j\le p-1,\\ i^3+j^3\ne 0}}(i^3+j^3)$.

We are also interested in a natural generalization of the original equation in quadratics to the cubics. For $p\ge 5$, we examine the nontrivial case when $p\equiv 1 \pmod 3$, and for $S=\{a^3 \mid a\in\mathbb{F}_p \text{ and } a\ne 0\}$ we evaluate the product $\prod_{\substack{i<j\in S\\ i+j\ne 0}}(i+j)$.

Sally Zhu

On the Smoothness and Regularity of the Chess Billiard Flow and Internal Waves

under the direction of Zhenhao Li

Abstract

The Poincar problem is a model of two-dimensional internal waves in stable-stratified fluid. The chess billiard flow, a variation of a typical billiard flow, drives the formation behind and describes the evolution of these internal waves, and its trajectories can be represented as rotations around the boundary of a given domain. We find that for sufficiently irrational rotation in the square, or when the rotation number $r(\lambda)$ is Diophantine, the regularity of the solution $u(t)$ of the evolution problem correlates directly to the regularity of the forcing function $f(x)$. Additionally, we show that when f is smooth, then u is also smooth. These results extend studies that have examined singularity points, or the lack of regularity, in rational rotations of the chess billiard flow. We also present numerical simulations in various geometries that analyze plateau formation and fractal dimension in $r(\lambda)$ and conjecture an extension of our results. Our results can be applied in modeling two dimensional oceanic waves, and they also relate the classical quantum correspondence to fluid study.