

RSI 2007

Abstracts

Mathematical Model of the Single Transferable Vote Election

Utsav Bhat

The Single Transferable Vote (STV) is a voting system used in some parts of Britain and Australia today. We model the voting system by randomly generating N candidates over $[0,1]$, and assigning areas closest to them. As N increases, the probability that the STV election and the plurality election will produce the same result decreases. In single-winner elections candidates at .5 have the highest chance of winning when there are two or three candidates. Candidates around .4 and .6 have the highest chance of winning as the number of candidates increases beyond 4. When the initial area before the first elimination is computed for an STV election, it tends to be lower than the initial area of a plurality election. When multiple winners are introduced, the number of relative maxima corresponds to the number of winners in the election. This is the case with both the STV and plurality election, but the maxima and minima are much more distinct for the STV election. When candidates are placed in multiple dimensions, both elections see the maxima of winners near the center. In the STV election, the difference is more distinct; candidates near the middle have a higher chance of winning, and candidates near the edge have a lower chance of winning.

Self-Reflective Comma-Free Codes

Alex Churchill

A major problem in coding theory is that of comma-free coding; that is, codes which can be received without the need for a letter used for word separation. The major difficulty is in finding bounds on the maximum number of words which can inhabit a dictionary. This paper introduces a new class called a self-reflective comma-free dictionary and proves a series of bounds on the size of that dictionary based upon word length and alphabet size. This paper also introduces other new classes such as self-swappable comma-free codes and comma-free codes in q dimensions and proves preliminary bounds for these classes.

On the Reducible Quintic Complete Base Polynomials

Alex Chen

Let $B(x) = x^m + b_{m-1}x^{m-1} + \dots + b_0 \in \mathbb{Z}[x]$. If every polynomial in $\mathbb{Z}[x]/(B(x)\mathbb{Z}[x])$ has a polynomial representative with coefficients in $S = \{0, 1, 2, \dots, |b_0| - 1\}$ then $B(x)$ is called a complete base polynomial. We prove that if $B(x)$ is a completely reducible quintic polynomial with distinct integer roots less than -1 , then B is a complete base polynomial. We then provide a program capable of determining whether an input $B(x)$ is a complete base polynomial.

Descriptive Complexity of Random Bit Strings

Benjamin Dozier

We investigate the first-order quantifier depth needed to distinguish non-isomorphic random bit strings $U_{p(n)}(n)$. We find tight bounds that hold almost surely for a wide range of functions $p(n)$, ranging from much less than $1/n$ (which means very sparse or no ones in the bit string), up to a constant value $c > 0$. The tight bounds range from as high as $\Theta(\log n)$ for $p(n)$ that give very sparse bit strings, down to $\Theta(\log \log n)$, for $p(n)$ that give bit strings with many ones and many zeroes.

On the Kakeya Conjecture in Three Dimensions

Philip Hu

Let \mathbb{F}_q^n be an n -dimensional vector space over a finite field of size q where q is the power of some prime. A *Besicovitch set* is a subset of \mathbb{F}_q^n that contains a line in every direction. The Kakeya conjecture in finite fields states that the minimum size of a Besicovitch set is bounded below by $C_n q^n$ for some constant C_n independent of q . In this paper, we take steps towards proving the Kakeya conjecture in three dimensions.

On Various Chip-Firing Games

Paul Kominers

We examine chip-firing games. We first prove formulae to compute the number of firings required to stabilize arbitrary maximized trees and cycles; we then combine those proofs to prove a formula for a specific type of chip-firing game on cacti. We then examine the candy-passing game as given by James Tanton. After showing that it is isomorphic to a restricted chip-firing game, we show that, with certain starting conditions, it will eventually oscillate with period 1.

Modeling Choices in the Traveler's Dilemma

Laney Kuenzel

The traveler's dilemma, invented by Kaushik Basu, is an example of a game for which traditional game theory concepts fail to explain observed behavior. In this paper we present three models which attempt to simulate how real people make choices when playing the traveler's dilemma. The naive assumption model, though very simple, fits one-shot experimental data the most closely of the models studied. The more complicated range splitting model provides a justification for the previously unexplained but experimentally observed direct relationship between R and average claims for some medium-sized R values. Furthermore, the range splitting model most successfully predicts experimental claims observed after several rounds of play. Finally, the active/passive model allows for different levels of rationality for different people. This model, unlike the others presented here, provides a possible reason for a player who understands the game to choose the maximum possible claim, something that Becker, Carter, and Naeve wrote that "we cannot explain at all."

Cake-Cutting with Locally Negative Preference Functions

Winston Luo

A number of algorithms exist for fair division of cakes. All previous procedures assume that the preference function of each player on all subsets of the cake is nonnegative. We consider the situation where preference functions are allowed to be negative and analyze how existing procedures function under this modification. We then present new algorithms for envy-free division of cake among three or four people, where preference functions can be both positive and negative. We present the additional result that bounded algorithms cannot ensure Pareto-optimal and proportional division between two people.

On the Problem of Finding the 27 Tropical Lines On A Tropical Cubic Surface

Ping Fung Ng

Tropical algebraic geometry is the geometry over the tropical semiring $(\mathbb{R} \cup \{\pm\infty\}, \max, +)$. In this paper, we give an introduction to tropical geometry. We also present analogous geometric objects in tropical geometry with their counterparts in classical projective geometry. New results include a construction of a smooth tropical cubic surface. In the final section, we present the problem of finding the 27 tropical lines on a smooth tropical cubic surface.

Counting Nodal Curves in CP^2 Via Floor Diagrams

Yasin Razlik

In this paper, we count the number of nodal curves in CP^2 with the help of floor diagrams. Mikhalkina's correspondence theorem tells us that the number of nodal curves with degree d in projective space CP^2 equals the number of floor diagrams with degree d . We also explain the Welschinger numbers and prove a theorem using floor diagrams. In addition, we calculate the number of floor diagrams with degree 3 and degree 4.

Subspace Intersection: Multi-dimensional Representation of Graphs

David Rolnick

A (d, e) -subspace intersection graph or (d, e) -SI graph is a graph which may be represented by a collection of e -dimensional affine subspaces in \mathbb{R}^d , where each subspace represents one vertex of the graph, and nonempty intersection between two subspaces denotes the existence of an edge between their corresponding vertices. We make progress towards a classification of $(3, 1)$ -SI graphs, in large measure by finding new examples of graphs lacking $(3, 1)$ -SI representations. Secondly, we find a graph lacking $(4, 2)$ -SI representations. Finally, we prove the equivalence of the sets of $(2e + k, e)$ -SI graphs for positive integral k .

On Search Techniques for Root-Unitary Polynomials

Caroline Suen

In this project we develop improvements on an algorithm which searches for $R(x)$ on a prescribed circle whose coefficients are rational numbers subject to congruence conditions. We do so by expanding an existing list of inequalities which make use

of power sums and Chebyshev polynomials, among other functions, to improve existing bounds on the coefficients of the polynomials. Using SAGE, an open-source computer algebra system, we test the inequalities and conclude that significant improvements occur.

Finding Rootless Matrices Using Jordan Normal Form

Anne Ye

In this paper, we use the Jordan normal forms of nilpotent matrices to find those that are rootless. We prove that an $n \times n$ matrix whose Jordan normal form has two Jordan blocks, neither of which has dimensions of $\lceil \frac{n}{2} \rceil \times \lceil \frac{n}{2} \rceil$, is rootless. We generalize this find to $n \times n$ matrices whose Jordan normal forms consist of three or more Jordan blocks, proving our results specifically for the case where one of the blocks has dimensions greater than 3×3 and the remaining blocks are 0's.

Elliptic Curves as Space Quartics

Qiaochu Yuan

Let Q be a space quartic of genus one given by the intersection of two quadric surfaces $\mathbf{XAX}^T = \mathbf{XBX}^T = 0$ in \mathbb{P}^3 ; then Q is birationally equivalent to an elliptic curve. In this paper we construct a normal form for all such Q , and in doing so find explicit transformations from Q to planar elliptic curves.

Decomposition of Algebraic Models of Quantum Information

Transference

Susan Zhang

To model the interactions between a quantum system and the environment, we attempt to decompose arbitrary mappings of quantum information. We find that to allow such decomposition to occur, additional dimensions must be added. The addition of dimensions causes an exponential increase in the number of equations and variables, and thus with the power of the average computer, the system is unable to be solved in a reasonable amount of time. We then propose other methods which could be undertaken to arrive at the desired solution.