# Program for Research in Mathematics, Engineering and Science



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Photo: Slava Gerovitch

# Fourteenth Annual Spring-Term PRIMES Conference May 19, 2024

## Fourteenth Annual Spring-Term PRIMES Conference

## Sunday, May 19, 2024

Room 2-190, MIT

## 9:15 am: Welcoming Remarks

• Dr. Slava Gerovitch, PRIMES Program Director

## 9:20-10:10 am: Session 1. Mathematics, Computational Biology, and Computer Science

- Sophia Hou, Henrick Rabinovitz, and Derek Zhao, "Gabriel's Theorem and the Subspaces Problem" (mentor Serina Hu)
- Elizaveta Rybnikova, "Loop Extrusion Dynamics at Chromatin Fountains" (mentor Dr. Aleksandra Galitsyna)
- Alan Song, "Characterizing Retry Policies for Microservice Applications" (mentor Yueying (Lisa) Li)

## 10:20-11:05 am: Session 2. Computer Science

- Sophia Lichterfeld, "An Analysis of the 2024 Web Monetization Landscape" (mentor Kyle Hogan)
- Boyan Litchev, "Improved Performance for Private Information Retrieval" (mentor Simon Langowski)
- Dongchen Zou, "Intersection Attack in a Non-uniform Setting" (mentor Simon Langowski)

## 11:20 am - 12:00 pm: Session 3. PRIMES STEP

- Aria Chen, Tyler Cummins, Rishi De Francesco, Jate Greene, Alexander Meng, Tanish Parida, Anirudh Pulugurtha, Anand Swaroop, and Samuel Tsui (PRIMES STEP Junior group), "Card Tricks and Information" (mentor Dr. Tanya Khovanova)
- Nikhil Byrapuram, Adam Ge, Selena Ge, Sylvia Lee, Rajarshi Mandal, Gordon Redwine, Soham Samanta, Daniel Wu, Danyang Xu, and Ray Zhao (PRIMES STEP Senior group), "Fibonacci Party Tricks" (mentor Dr. Tanya Khovanova)

## 12:30-1:40 pm: Session 4. PRIMES Circle

- Marisa Gaetz & Mary Stelow, PRIMES Circle coordinators, Welcoming remarks
- Ronni Chang & Paige Zhu, "Introduction to Computation Theory" (mentor Zoe Xi)
- Mohamed Fofana Jr. & Tanesha Nixel, "Game Theory" (mentor Sam Packman)
- Joshua Pité & Yiying Zhong, "Overview of the RSA Cryptosystem" (mentor Honglin Zhu)
- Ayla Lin, Kelsey Liu, & Vedika Soneji, "Intersecting Parallel Lines: Projective Geometry and its Applications" (mentor Luis Modes)

## 1:55-2:55 pm: Session 5. PRIMES Circle

- Nina Lee & Yurie Lee, "Introduction to Probability" (mentor Maggie Lin)
- Youdi (Ivy) Yao & Julia Zhou, "A Special Case of Zykov's Theorem and the Shifting Method" (mentor Tomasz Slusarczyk)
- Nairi Davidian & Winnie Gong, "Knots, Genus, and Surfaces" (mentor Yuyuan Luo)
- Hwiseo (Irene) Choi, Bang Tam Ngo, & Larine Ouyang, "Classification of Cyclic Subgroups with the Fundamental Theorem of Cyclic Groups" (mentor Zhao Yu Ma)

## 3:10-4:25 pm: Session 6. PRIMES Circle

- Neel Chattopadhyay & Nahum Linhart, "Diophantine Equations" (mentor George Shaker)
- Ladia Khaing & Ada Kilian, "The Szemeredi-Trotter Theorem" (mentor Paige Dote)
- Aleah He, Bhavika Kalia, & Laurelyn Newsome, "Exploring Knot Theory: Surfaces and Summation" (mentor Carlos Á. Alvarado)
- Quinten Jin & Emre Kocaman, "Combinatorics: An Exploration of Counting Principles and Their Applications" (mentor Sean Li)
- Olivia Chen & Natalie Han, "The Math Behind 'Spot It!'" (mentor Katherine Tung)

#### 2024 PRIMES CONFERENCE ABSTRACTS

Session 1. Mathematics, Computational Biology, and Computer Science

#### Sophia Hou, Henrick Rabinovitz, and Derek Zhao

Gabriel's Theorem and the Subspaces Problem

#### Mentor: Serina Hu

This presentation aims to showcase a connection between quiver representations, Gabriel's Theorem, and the subspaces problem. We will begin by giving an overview of the subspaces problem, along with short solutions to the one subspace and pairs of subspaces problems. After this, we will introduce the concept of quivers and their representations as a framework for the more complex triples of subspaces problem. This will lead us to Gabriel's Theorem, which will relate the indecomposable representations of a quiver with positive roots of a graph associated to that quiver. Finally, we will utilize Gabriel's theorem to solve the triples of subspaces problem.

#### Elizaveta Rybnikova

#### Loop Extrusion Dynamics at Chromatin Fountains

#### Mentor: Dr. Aleksandra Galitsyna

DNA in the nucleus of eukaryotic cells is a compact structure that forms a variety of local structures – which appear as domains, fountains, and stripes in Hi-C maps – responsible for the regulatory interactions between promoters and enhancers. In mammals, these structures are known to be formed by the mechanism of loop extrusion. In this process, a motor protein binds to DNA, connecting two regions of DNA together in a loop. This loop is processively extruded by the motor protein cohesin until it falls off. Cohesin traces manifest themselves in the Hi-C maps as a fountain. In our previous research, we focused on studying fountains in the Hi-C maps and confirmed their existence in mammalian cells. In the current follow-up, we focus on simulating cohesin movement and establishing the parameters where cohesin produces detectable traces in Hi-C and single-cell Hi-C maps. Here, we present the study of co-localization dynamics of genomic regions mediated by cohesin bonds that lead to a long-lasting trace and increased probability of promoter-enhancer interactions.

#### Alan Song

#### Characterizing Retry Policies for Microservice Applications

#### Mentor: Yueying (Lisa) Li

Microservices have emerged as a prevalent architectural paradigm, particularly in large-scale applications, as a replacement for monolithic systems. The distributed nature of microservices, however, makes them more susceptible to failures, necessitating robust retry mechanisms for fault tolerance. Despite their usefulness, retry mechanisms have been documented in production environments as a common cause of metastable failures. Such failures have severe impact, are difficult to predict, and are sustained by feedback loops even after an initial trigger is resolved. To better understand the relationship between retry mechanisms and metastable failures, we characterize the efficacy of different retry policies in mitigating metastable failures and their impact on end-to-end latency in microservice applications. Our research aims to shed light on the intricate relationship between retry mechanisms and metastable failures in microservices. By understanding the underlying causes and effects, we strive to develop more resilient and efficient retry strategies that will enhance the reliability and performance of microservice architectures in real-world deployments. Session 2. Computer Science

#### Sophia Lichterfeld

#### An Analysis of the 2024 Web Monetization Landscape Mentor: Kyle Hogan

W3C's Web Monetization (WM) API offers users the ability to compensate content creators online by continuously streaming micropayments to the website owner while viewing a page. While WM could be a feasible alternative to advertisements or subscriptions, it has not yet been widely adopted by websites. Rates of WM adoption were tracked from 2019 to 2021 but have not been evaluated for the past several years. To implement WM, website owners must add a meta tag with a payment pointer directing the money to their online wallet into their page's HTML head. Using the presence of the meta tag as an indicator of WM adoption, we built a web scraper to determine the current WM adoption rate in 2024. To expand our adoption rate results, we analyzed a dataset curated by HTTP Archive through Google's BigQuery database. We further assessed the breakdown of wallet providers, the distribution of site popularity, and the comparison to these metrics in 2021. We hope our findings will fill this data gap and better inform approaches to increasing widespread WM adoption.

#### **Boyan Litchev**

#### Improved Performance for Private Information Retrieval Mentor: Simon Langowski

By allowing users to retrieve items from a database without revealing which item was retrieved, Private Information Retrieval (PIR) has enabled recent advances in anonymous communication, private streaming, and more. However, PIR is very computationally expensive, and is fundamentally limited to having a computational cost that scales linearly with the size of the database, limiting the scale of protocols that use it to millions of users. By adjusting the procedure for gadget inversions, a key step in the homomorphic multiplications used in PIR, we achieve a 30% speedup over existing state-of-the-art PIR protocols and similarly reduce network costs.

#### Dongchen Zou

#### Intersection Attack in a Non-uniform Setting

#### Mentor: Simon Langowski

Recently consumer demand for privacy has spurred growth in private messaging systems. However, formally, privacy degrades in such systems when users log on and off: this change of status exposes the ongoing conversations. Intersection attacks (also known as statistical disclosure attacks) use messaging patterns or liveness information to reconstruct relationships, deanonymize users, and track user behaviors. Prior attacks assume users have an underlying uniform communication pattern for simplicity, leaving the question open of how effective such attacks would be in a non-uniform real world. We observe that effects like clustering in real social graphs and correlation between repeated conversations change the behavior and potential of such attacks. This talk provides a new approach that can consider some of these additional factors by constructing a polynomial to determine the social graph. We provide an analysis of the performance, accuracy, and convergence rate of our attack. Our attack applies to many existing anonymous communication systems, and our technique can be extended to incorporate additional factors.

#### SESSION 3. PRIMES STEP

#### Aria Chen, Tyler Cummins, Rishi De Francesco, Jate Greene, Alexander Meng, Tanish Parida, Anirudh Pulugurtha, Anand Swaroop, and Samuel Tsui (PRIMES STEP Junior group)

Card Tricks and Information

#### Mentor: Dr. Tanya Khovanova

How are Hogwarts and the MIT PRIMES STEP the same? They both teach magic! This presentation will be on card tricks based on information theory. During the presentation, you will learn four card tricks: the Cheney 5-card trick, the Kleber and Vakil 5-card trick, the Mulcahy 4-card trick, and our new 3-card trick! We hope you'll join us on this magical journey!

#### Nikhil Byrapuram, Adam Ge, Selena Ge, Sylvia Lee, Rajarshi Mandal, Gordon Redwine, Soham Samanta, Daniel Wu, Danyang Xu, and Ray Zhao (PRIMES STEP Senior group)

#### Fibonacci Party Tricks

#### Mentor: Dr. Tanya Khovanova

Magic tricks are incredibly fun, and often they involve math, and these are no different. The following magic trick is the motivation for this presentation. While the audience writes the first ten terms of a Fibonacci-like sequence (the sequence following the same recursion as the Fibonacci sequence) the magician calculates the sum of these ten terms very fast by multiplying the 7th term by 11. This trick is based on divisibility properties of partial sums of the Fibonacci sequence. We will discuss the maximum Fibonacci number that divides the sum of the Fibonacci numbers 1 through n.

We will also discuss connections between trigonometric identities and Fibonacci identities. Conway and Ryba in 2013 studies these connections and came up with a set of rules to convert a trigonometric identity to Fibonacci identities and vice versa. We generalize these rules for Lucas Sequences. Session 4. PRIMES Circle

#### Ronni Chang & Paige Zhu

Introduction to Computation Theory

#### Mentor: Zoe Xi

In this presentation, we present the fundamental concepts computation theory, which comprises automata, computability, and complexity theory. We will introduce models of computation, specifically deterministic finite automata and Turing machines. We will also discuss time complexity and big-O and small-o notation, and the importance of polynomial time algorithms.

#### Mohamed Fofana Jr. & Tanesha Nixel

Game Theory

#### Mentor: Sam Packman

In this presentation, we will cover combinatorial games with an example of SOS and a game tree, normal play games with the types of positions, and an example of Domineering, and we will be explaining impartial games with binary expansion, examples of Nim, and balanced and unbalanced positions.

#### Joshua Pité & Yiying Zhong

Overview of the RSA Cryptosystem

#### Mentor: Honglin Zhu

The RSA cryptosystem is one of the earliest and most well-known protocols for public key cryptography. In this presentation, we give a historical and technical overview of the RSA cryptosystem. We briefly outline the history of crytography, introduce mathematical foundations for the RSA, explain its algorithm, discuss security implications, and give a demonstration of our Python implementation of RSA.

#### Ayla Lin, Kelsey Liu, & Vedika Soneji

Intersecting Parallel Lines: Projective Geometry and its Applications

#### Mentor: Luis Modes

In traditional Euclidean Geometry, parallel lines never intersect. In our perception of the world, however, parallel lines appear to converge at vanishing points infinitely far away. Projective geometry explores this possibility; at its core, this type of geometry examines the properties of points and lines, and how they behave after being subjected to transformations of perspective. In this presentation, we present an exploration of analytic projective geometry, its sub-geometries, projective transformations, and its very useful applications in art, animation, and game design. Session 5. PRIMES Circle

#### Nina Lee & Yurie Lee

Introduction to Probability

#### Mentor: Maggie Lin

Probability is a mathematical term used to talk about the likelihood of something happening, and it is the ability to understand and predict an outcome. In our presentation we will be talking about conditional probability, Bayes' Theorem, and independent events with real-life examples.

#### Youdi (Ivy) Yao & Julia Zhou

A Special Case of Zykov's Theorem and the Shifting Method

#### Mentor: Tomasz Slusarczyk

Zykov's theorem tells how many cliques of size k can a graph without a clique of size l have. We prove this theorem for k = 3 and l = 5. We use two techniques: graph-theoretic symmetrization introduced by Zykov himself and the shifting method of proving inequalities.

#### Nairi Davidian & Winnie Gong

Knots, Genus, and Surfaces

#### Mentor: Yuyuan Luo

Knots are closed curves embedded in three-dimensional space, and can be form by looping a piece of string around itself in various ways and then gluing the ends together. In our talk we discuss a particular invariant of knots, the genus, which can help us tell knots apart. We define the genus of a knot via a construction of Seifert surfaces and give formulas in terms of the Euler characteristic. We conclude by discussing the genus of two particular classes of knots: twist knots and prime knots.

#### Hwiseo (Irene) Choi, Bang Tam Ngo, & Larine Ouyang

Classification of Cyclic Subgroups with the Fundamental Theorem of Cyclic Groups

#### Mentor: Zhao Yu Ma

Group theory serves as a foundational part of modern mathematics, offering a profound understanding of the structure and relationships of various mathematical systems. This presentation delves into the concepts of cyclic group subgroups using the Fundamental Theorem of Cyclic Groups as its cornerstone. Beginning with the definition of groups, subgroups, and cyclic groups, the presentation explores their various properties, leveraging them to introduce the Fundamental Theorem of Cyclic Groups, which establishes the uniqueness and cyclical nature of subgroups within cyclic groups. Applying the theoretical framework provided by the Fundamental Theorem of Cyclic Groups, this presentation explains further on identifying cyclic groups, overall providing insights into the diverse application of the subgroups of cyclic groups, contributing to a deeper understanding of group theory. SESSION 6. PRIMES CIRCLE

#### Neel Chattopadhyay & Nahum Linhart

Diophantine Equations

#### Mentor: George Shaker

This presentation discusses Diophantine equations: equations that express a specific natural number. This talk will primarily cover over one specific Diophantine Equation: The Sum of Two Squares. We will discuss and prove what natural numbers can be expressed as a sum of two integer squares.

#### Ladia Khaing & Ada Kilian

The Szemeredi-Trotter Theorem

#### Mentor: Paige Dote

Given a set of points, P, and lines, L, in Euclidean space, how large can  $\{(p, \ell) \in P \times L : p \in \ell\}$ , the number of incidences between points and lines be? The sharp bound is known as the Szemeredi-Trotter theorem. In this presentation, we will outline how one utilizes graph theory and probability to obtain this powerful result in geometric combinatorics.

#### Aleah He, Bhavika Kalia, & Laurelyn Newsome

Exploring Knot Theory: Surfaces and Summation

#### Mentor: Carlos Á. Alvarado

In our presentation, we will delve into the fascinating realm of knot theory, focusing on the intricate relationship between knots and surfaces. We will explore the computation of surfaces of genus g and the fundamental concept of Seifert surfaces. Additionally, we will discuss the implications of knot summation, including Seifert's algorithm and the genus sum formula.

#### Quinten Jin & Emre Kocaman

Combinatorics: An Exploration of Counting Principles and Their Applications

#### Mentor: Sean Li

This presentation is about combinatorics, focusing on several principles and their applications in solving complex counting problems. Through the usage of the Principle of Inclusion and Exclusion (PIE), Catalan numbers, and generating functions, we explore a variety of problems ranging from language overlaps in multilingual classes to domino tiling puzzles. The initial section uses PIE to dissect a classroom language scenario, providing a foundational understanding of counting without overestimation. Then, we will talk about Catalan numbers to explain counting structures maintaining non-negative cumulative sums and path constraints. The presentation also tackles the Domino Tiling Problem, introducing a recursive relationship that lead to the Fibonacci sequence to determine tiling configurations for larger boards. Lastly, we expand on generating functions, a powerful tool for encoding sequences and solving combination problems systematically.

#### Olivia Chen & Natalie Han

The Math Behind "Spot It!"

#### Mentor: Katherine Tung

Groups are a type of algebraic structure in abstract algebra with basic properties that give it a wide variety of applications in mathematics and science. For instance, in geometry, groups are essential to understanding projective geometry, which was historically used in the context of art as artists searched for the principles of "projecting" the three-dimensional objects they wanted to depict onto their two-dimensional canvases. Furthermore, the unique properties of groups and projective geometry allow it to model many other interesting scenarios, such as how to design a Spot It! deck. They also can further extend their use even into combinatorics through Steiner systems. Although seemingly simple, many complex concepts and situations can be broken down using groups, making them fundamental to many fields of study. In this talk, we discuss matrix groups, projective space, projective geometry over a finite field, the math behind Spot It!, and Steiner systems.