Intersection Attack in Non-Uniform Setting

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Introduction

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- Messages themselves? They are normally encrypted and hard to obtain.
- Activity pattern like logging on and off?
- In this talk, we will explore how such information can be used to learn user connections.

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- People tend to talk based on the number of common interests.
- If talking previously, it is more likely for them to talk later.

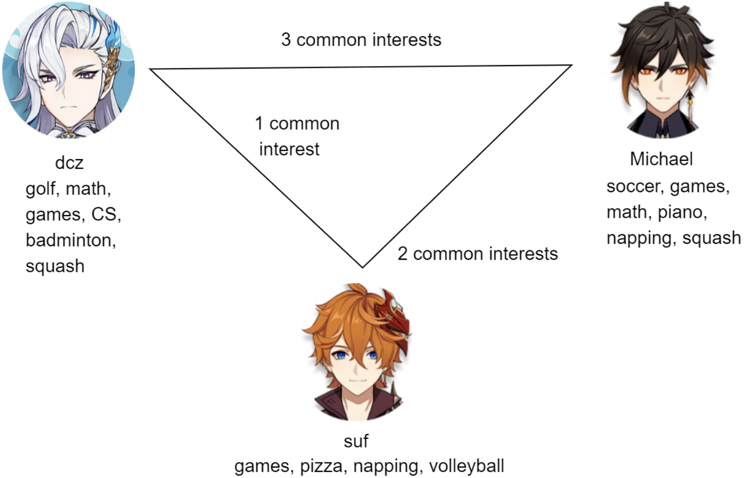
An Example of Social Media

dcz
golf, math,
games, CS,
badminton,
squash

Michael
soccer, games,
math, piano,
napping, squash

suf
games, pizza, napping, volleyball

An Example of Social Media



Eavesdropper's Observation

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The eavesdropper gets to see all the people who are online in a period of time called an epoch.

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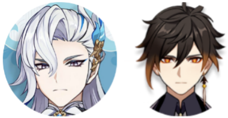
Epoch 1



Epoch 2

Epoch 3

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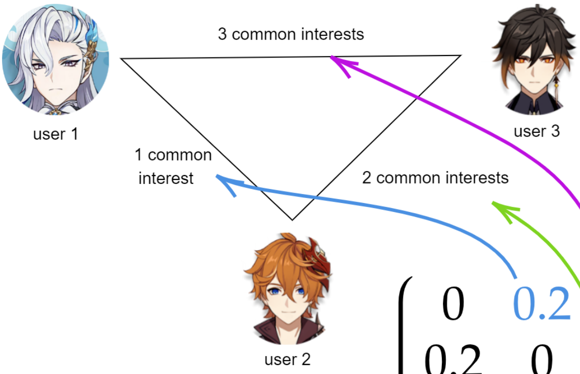
- For each pair $[\mathrm{i}, \mathrm{j}]$, the larger the intersection, the more probable it is that they talk.
- We will denote probability that i and j talk in an epoch with $\mathrm{A}[\mathrm{i}, \mathrm{j}]$

Probability Matrix A for the Example Graph



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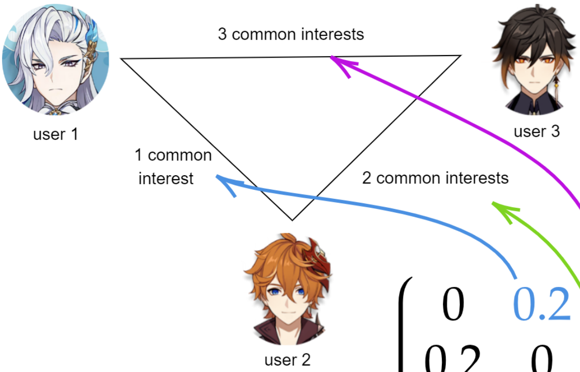

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The eavesdropper is trying to figure out the probability matrix.

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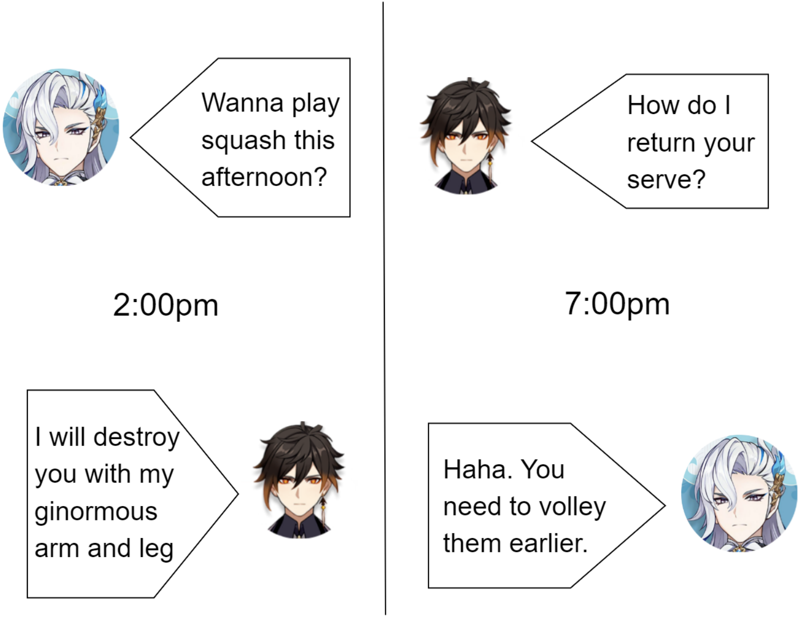
- If i and j talked in the previous epoch, it is more likely for them to keep talking in this epoch.
- How about $A[i, j] + \delta$ if i and j talked in last epoch?
- This change is apparently temporary and will vanish once $A[i, j] + \delta$ flips to tail

Correlation for the Example Graph



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- User i having at least one conversation with someone other than j
- $g(i,j) = 1 - \prod_{k \notin j}(1 - \mathrm{A}[i,k])$
- $$\mathrm{F}_{[\mathrm{i,j}]}(A) = \mathrm{A}[\mathrm{i,j}] + (1 - \mathrm{A}[\mathrm{i,j}]) \cdot g(i,j) \cdot g(j,i)$$

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- What is this probability, call it $C_{[\mathrm{i,j}]}$, as observed by the eavesdropper?
- Number of times divided by total number of epochs!

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Now we want to consider the entire graph. Let $\mathrm{F}(\mathrm{A}) = \sum \mathrm{F}_{[\mathrm{i},\mathrm{j}]}(\mathrm{A})$ and $C = \sum C_{[\mathrm{i},\mathrm{j}]}$

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- Why does it work?
- $$\lim_{t \to \infty} C_t = \mathrm{F}(\mathrm{A})$$

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- We are interested in the rate of convergence.

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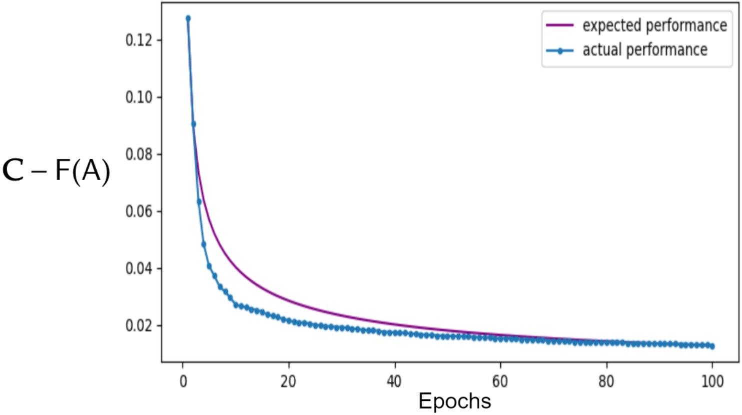
- If I double the number of epochs given, the new difference should be $\frac{1}{\sqrt{2}} \approx 70.7\%$ of the previous difference.

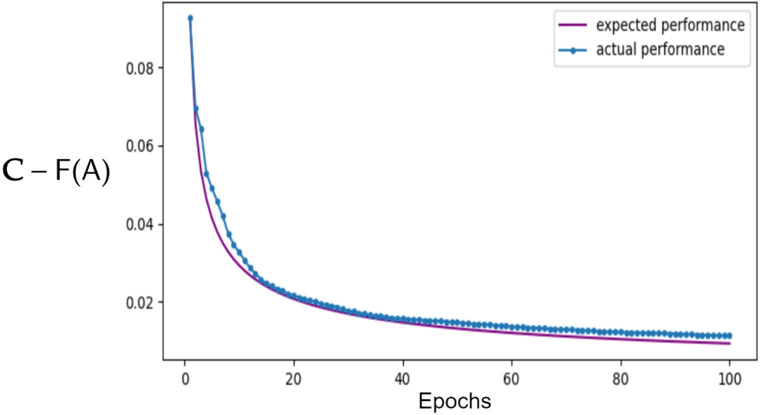
Results

Only clustered



Results

Clustered and Correlation



Future Work

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Acknowledgement

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- You guys for coming to my talk!