Quaternion-Based Analytical Inverse Dynamics for the Human Body

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under the mentorship of David Darrow

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Inverse Dynamics for the Human Body

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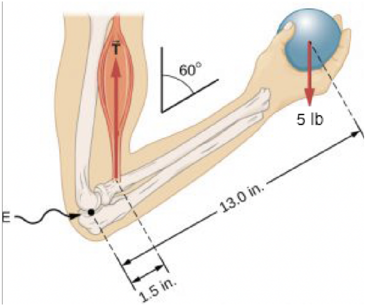

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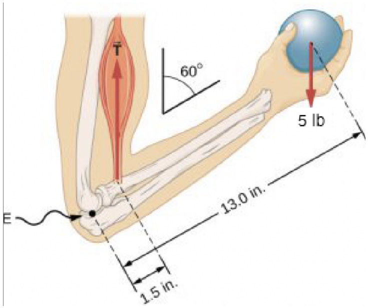
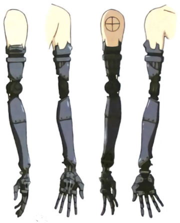


- It's hard to measure these internal dynamics directly, so we need numerical models.

openoregon.pressbooks.pub

Why We Should Care



Prosthetic Design

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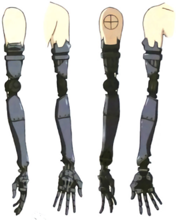

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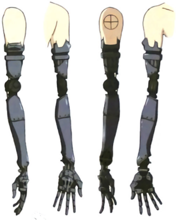

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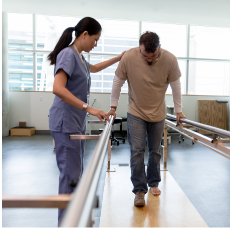

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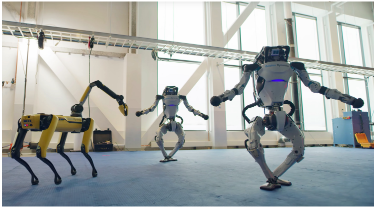

Human-Inspired Robots

reddit.com/r/VioletEvergarden, static01.nyt.com, engadget.com

Overview

1. Existing methods and models

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2. Theoretical background

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2. Theoretical background
3. Our novel method

Some Terminology

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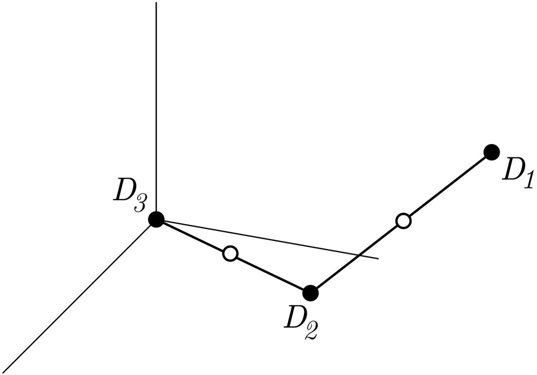
ICS, SCS: The *inertial coordinate system* (ICS) and *segment coordinate system* (SCS) are the global and segment-specific coordinate axes, respectively.

The Traditional Way of Doing Things

Normally, we start off with a diagram like this:



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However, there are some shortcomings to this method.

The Orientation Problem

- The usual way of tracking the orientation of each segment uses **Euler angles**

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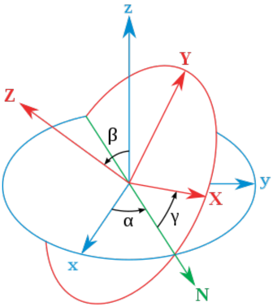
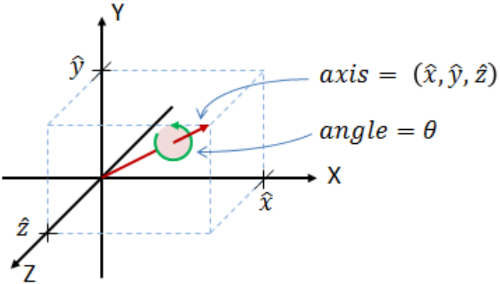
- This structure suffers from singularities, or **gimbal lock**



en.wikipedia.org/wiki/Euler_angles

The Quaternion Solution

- Quaternions define a unique transformation and orientation in terms of an axis and an angle



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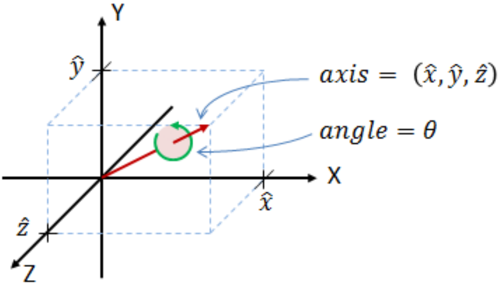

- The corresponding quaternion to such a transformation is

$$q = \cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)\vec{\boldsymbol{u}}$$

danceswithcode.net/engineeringnotes/quaternions/

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  - No three coordinate system can do this, for geometric reasons
- Quaternions are also very convenient for rotating vectors, as we simply **conjugate** them
  - Conjugating a vector simply entails multiplying it by the quaternion and its conjugate, in order: $qvq*$

The One Step Inverse Dynamic Method

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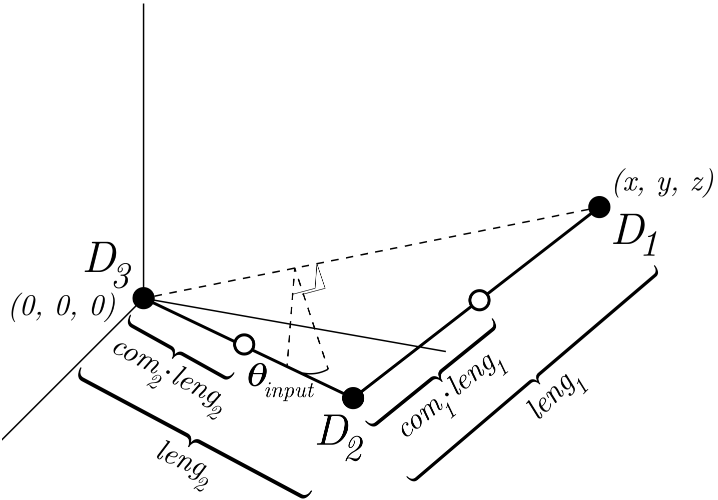
- Screws are really just a concatenation of two specific kinds of vectors: a linear one and a related angular one
- In our case, we use the **wrench**, which is a force and moment vector combined.
- Screw algebra gives a single step method of calculating the wrench at each subsequent joint (Dumas, 2004):

$$\begin{bmatrix} \vec{F}_i \\ \vec{M}_i \end{bmatrix} = \begin{bmatrix} m_i \mathbf{E}_{3\times3} & \mathbf{0}_{3\times3} \\ m_i \tilde{\mathbf{c}}_i & \mathbf{I}_i \end{bmatrix} \begin{bmatrix} \vec{a}_i - \vec{g} \\ \vec{\alpha}_i \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{3\times1} \\ \vec{\omega}_i \times \mathbf{I}_i \vec{\omega}_i \end{bmatrix} + \begin{bmatrix} \mathbf{E}_{3\times3} & \mathbf{0}_{3\times3} \\ \tilde{\mathbf{d}}_i & \mathbf{E}_{3\times3} \end{bmatrix} \begin{bmatrix} \vec{F}_{i-1} \\ \vec{M}_{i-1} \end{bmatrix}$$

The Basic Model: A Diagram and a Brief Explanation



The Basic Model: Equations

We assume for now that the arm is not in motion. Then in this framework, all moments and forces can be obtained from a sequence of matrix products:

$$\text{Wrist:} \quad \begin{bmatrix} \vec{\boldsymbol{F}}_1 \\ \vec{\boldsymbol{M}}_1 \end{bmatrix} = \begin{bmatrix} \overrightarrow{\boldsymbol{m_0 g}} \\ \vec{0} \end{bmatrix}$$

$$\text{Elbow:} \quad \begin{bmatrix} \vec{\boldsymbol{F}}_2 \\ \vec{\boldsymbol{M}}_2 \end{bmatrix} = \begin{bmatrix} m_1 \mathbf{E}_{3\times 3} & \mathbf{0}_{3\times 3} \\ m_1 \tilde{\mathbf{c}}_1 & \mathbf{I}_1 \end{bmatrix} \begin{bmatrix} -\mathbf{g} \\ 0 \end{bmatrix} + \begin{bmatrix} \mathbf{E}_{3\times 3} & \mathbf{0}_{3\times 3} \\ \tilde{\mathbf{d}}_1 & \mathbf{E}_{3\times 3} \end{bmatrix} \begin{bmatrix} \vec{\boldsymbol{F}}_1 \\ \vec{\boldsymbol{M}}_1 \end{bmatrix}$$

$$\text{Shoulder:} \quad \begin{bmatrix} \vec{\boldsymbol{F}}_3 \\ \vec{\boldsymbol{M}}_3 \end{bmatrix} = \begin{bmatrix} m_2 \mathbf{E}_{3\times 3} & \mathbf{0}_{3\times 3} \\ m_2 \tilde{\mathbf{c}}_2 & \mathbf{I}_2 \end{bmatrix} \begin{bmatrix} -\mathbf{g} \\ 0 \end{bmatrix} + \begin{bmatrix} \mathbf{E}_{3\times 3} & \mathbf{0}_{3\times 3} \\ \tilde{\mathbf{d}}_2 & \mathbf{E}_{3\times 3} \end{bmatrix} \begin{bmatrix} \vec{\boldsymbol{F}}_2 \\ \vec{\boldsymbol{M}}_2 \end{bmatrix}$$

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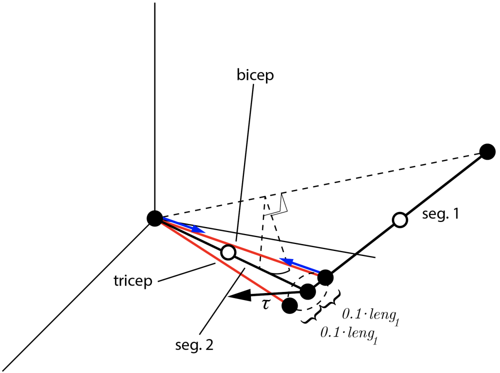
The middle wrench is gone, as are both acceleration vectors:

$$\begin{bmatrix} \vec{F}_i \\ \vec{M}_i \end{bmatrix} = \begin{bmatrix} m_i \mathbf{E}_{3\times3} & \mathbf{0}_{3\times3} \\ m_i \tilde{\mathbf{c}}_i & \mathbf{I}_i \end{bmatrix} \begin{bmatrix} \cancel{\vec{a}}_i - \vec{g} \\ \cancel{\vec{\alpha}}_i \end{bmatrix} + \cancel{\begin{bmatrix} \mathbf{0}_{3\times1} \\ \vec{\omega}_i \times \mathbf{I}_i \vec{\omega}_i \end{bmatrix}} + \begin{bmatrix} \mathbf{E}_{3\times3} & \mathbf{0}_{3\times3} \\ \tilde{\mathbf{d}}_i & \mathbf{E}_{3\times3} \end{bmatrix} \begin{bmatrix} \vec{F}_{i-1} \\ \vec{M}_{i-1} \end{bmatrix}$$

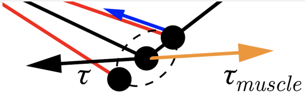
Adding Musculature

Before adding additional segments, we introduce a framework for incorporating muscles into the model.



Each muscle is treated as a tension between two fixed endpoints.

Changes to the Algorithm



We now calculate the wrenches twice.

1. Muscle-free wrench calculation

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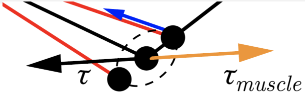

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1. Muscle-free wrench calculation
2. Find muscle tension

$$\begin{bmatrix} \vec{\boldsymbol{F}}_i \\ \vec{\boldsymbol{M}}_i \end{bmatrix}_m = \sum_{\substack{1 \le j \le M, \\ n_d(j)+1=i}} \begin{bmatrix} \vec{\boldsymbol{u}}_j \cdot \mu_j \\ \vec{\boldsymbol{d}}_{n_d(j)} \cdot (1 - r_d(j)) \times \vec{\boldsymbol{u}}_j \cdot \mu_j \end{bmatrix}$$

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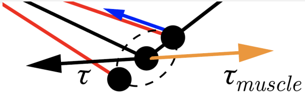

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3. Recalculate the wrenches with a modified equation:

$$\begin{bmatrix} \vec{F}_{i+1} \\ \vec{M}_{i+1} \end{bmatrix} = \begin{bmatrix} m_i \mathbf{E}_{3\times 3} & \mathbf{0}_{3\times 3} \\ m_i \tilde{\mathbf{c}}_i & \mathbf{I}_i \end{bmatrix} \begin{bmatrix} -\mathbf{g} \\ 0 \end{bmatrix} + \begin{bmatrix} \mathbf{E}_{3\times 3} & \mathbf{0}_{3\times 3} \\ \tilde{\mathbf{d}}_i & \mathbf{E}_{3\times 3} \end{bmatrix} \begin{bmatrix} \vec{F}_i \\ \vec{M}_i \end{bmatrix} + \begin{bmatrix} \vec{F}_{i+1} \\ \vec{M}_{i+1} \end{bmatrix}_m$$

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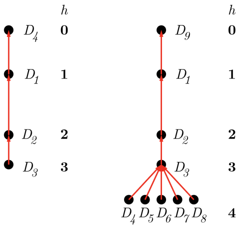
- There are exponentially more possible ways to hold an object as segments increase in number

The New Indices

To allow for the implementation of a nonlinear structure, we use a tree:



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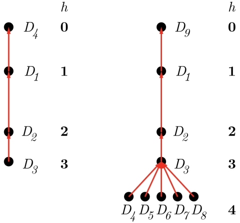

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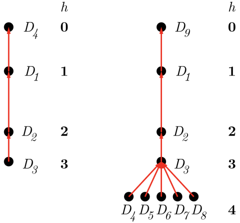

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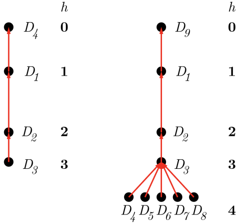
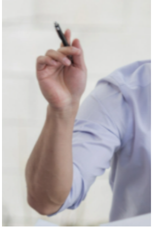


- Our indices are now simply a labelling serving as a way to distinguish points
- We create a hierarchy based on the number of edges between the shoulder and each point.
- Calculations now iterate along each hierarchy number

Orienting the Segments



- Each additional segment we add to the model creates more degrees of freedom

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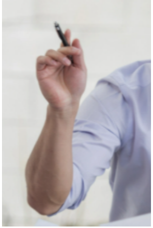

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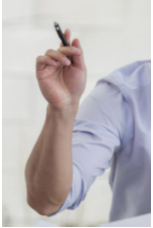
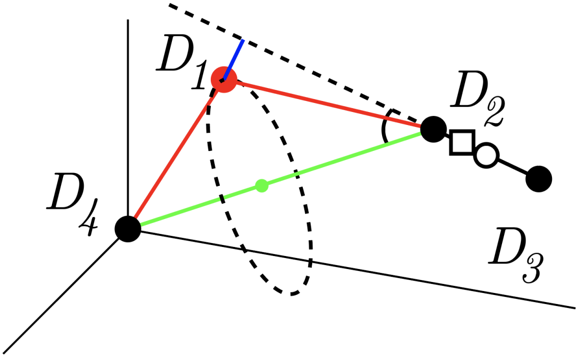


- Each additional segment we add to the model creates more degrees of freedom
- The human body usually uses the same orientation to hold an object
- We create a rudimentary way of predicting how the arm will naturally position itself

Simu Liu Stock Photo

Orienting the Segments

The gist of it is that we find whatever orientation minimizes bending at the wrist:



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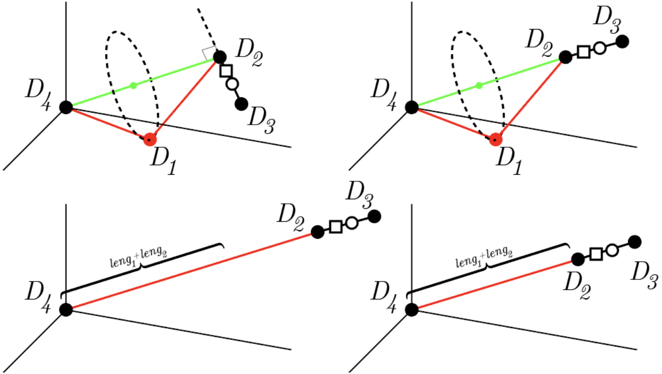

Recap

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- Screw algebra for efficiency

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- Implementation of the hand

Acknowledgements

I'd like to thank David Darrow as my mentor for the PRIMES project, Dr. Daniel Nolte for suggesting the topic for this project, as well as Dr. Tanya Khovanova, Prof. Pavel Etingof, and Dr. Slava Gerovitch for running the PRIMES research program.