Thermocapillary Modulation of Fluidic Lenses in Microgravity

Rishabh Das
Mentor: Dr. Valeri Frumkin

Stuyvesant High School

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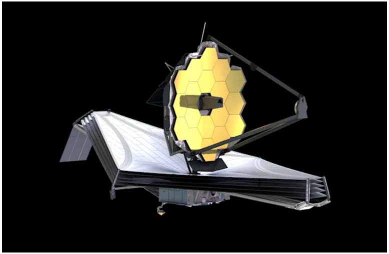
Motivation

The James Webb telescope is launched into space, and then is supposed to unfold.



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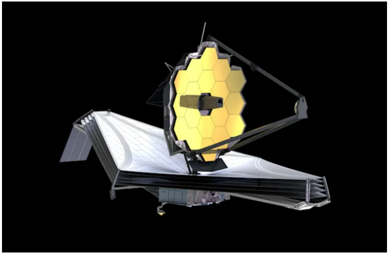

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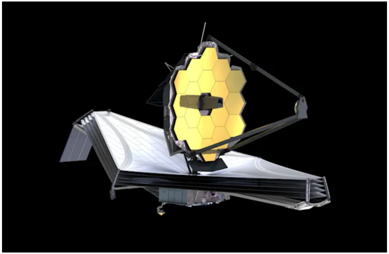

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What if we can form the lens in space?

Using Liquids

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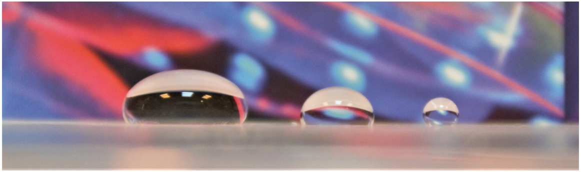
They are also more cost-efficient.

Capillary Length



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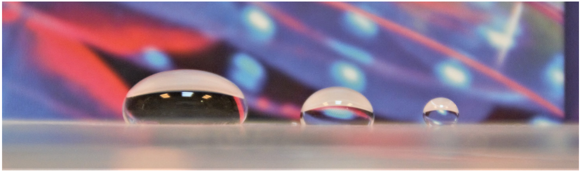

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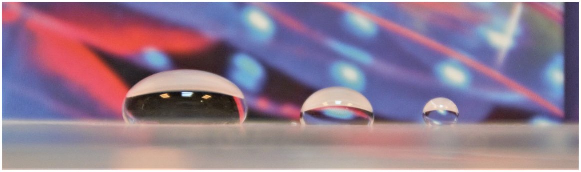

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Definition (Capillary Length)

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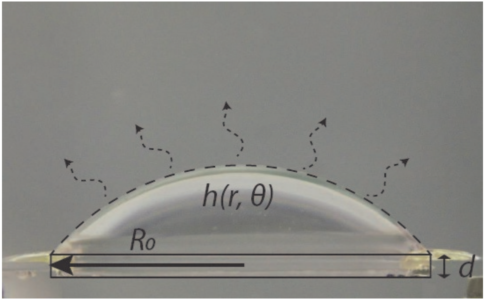
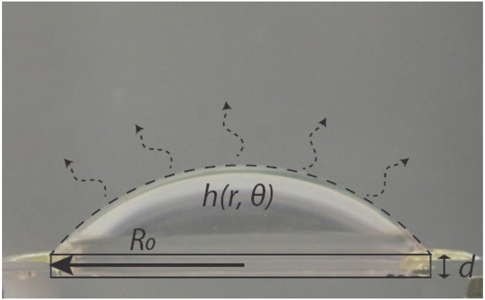
To make ℓ large, we can either make $\Delta\rho \approx 0$ or $g \approx 0$.

The Setup





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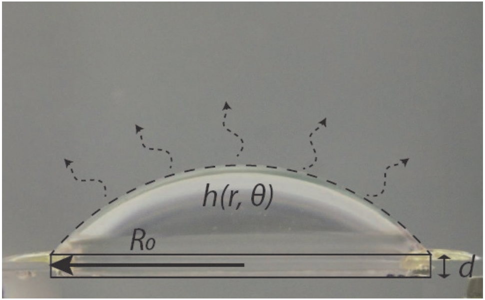

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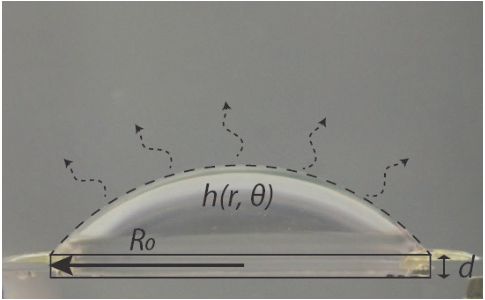

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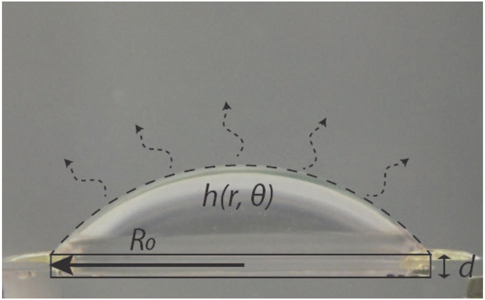

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With constant surface tension, the shape of the lens is spherical. We want to slightly deform this shape to correct for spherical aberrations.

First Method

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We just have to minimize the interfacial energy, i.e. the product of the surface tension and surface area.

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$$G(r, \theta) = \left(\gamma_0 \cdot f(r, \theta) \sqrt{1 + h_r^2 + \frac{1}{r^2} h_\theta^2} + \lambda h \right) r,$$

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In order to minimize this, we can use the Euler-Lagrange Equation:

$$\frac{\partial G}{\partial h} - \frac{d}{dr}\frac{\partial G}{\partial h_r} - \frac{d}{d\theta}\frac{\partial G}{\partial h_\theta} = 0.$$

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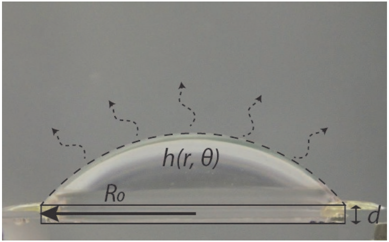

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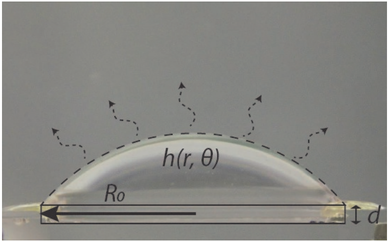

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In our situation, ε is very small. This means any term with ε in it is negligible. If we ignore such terms, our system is greatly simplified!

Our Results

The final equation we get is the following.

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Surface Equation (Das & Frumkin)

We have

$$PR^2 - (R^2 F_R H_R + F_\Theta H_\Theta) - F \cdot (H_{RR} R^2 + R H_R + H_{\Theta\Theta}) = 0,$$

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We choose $F(x) = 1 - \beta x^2$.

Our Results

The solution to the system is

$$H(x) = C_2 + C_1 \log x \mp \frac{(1 + 2\beta C_1)\log(1 - \beta x^2)}{4\beta}.$$

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For different functions F, we would get different solutions for H. We chose this F because it allows us to solve for H analytically. For more complex functions F, such as functions that are Θ dependent, we could construct more complex lenses, but we may not be able to solve for them analytically.

Graphs

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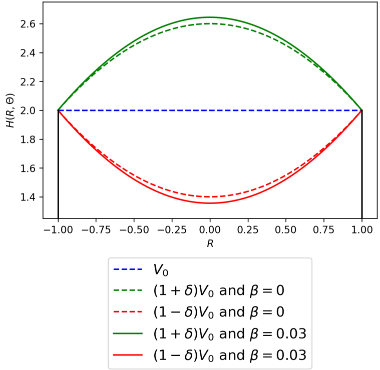

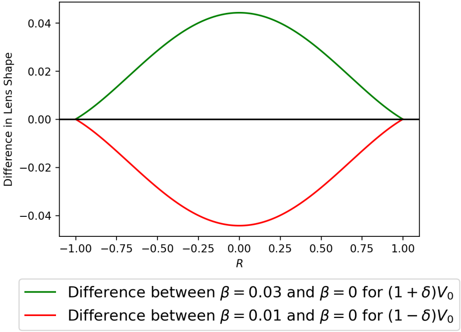

Marangoni Effect

Liquids tend to flow from high temperature to low temperature. This is called the Marangoni Effect.

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These flows can affect the shape of the lens in thin films. Since the temperature gradient remains the same, the flows will be constant.

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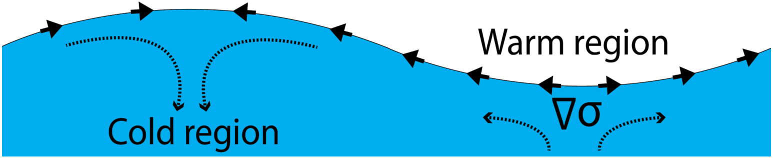

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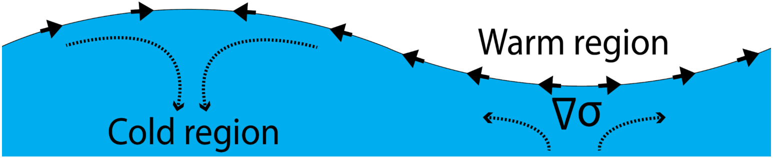

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- my parents and my brother

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