
Surfaces in Knot Theory

— By: Ben Ratin —

Primes Circle

Background

My name is Ben Ratin.

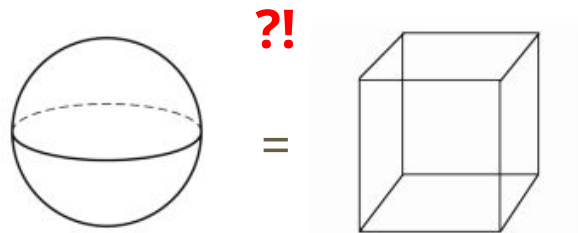
I am a rising Junior at Newton North High School.

My interests include Mathematics and several Sciences including Physics and Chemistry.

My mentor was Kenneth Kox

Background

- Presenting: the Knot Theory and how it relates to Surfaces around us
- We can use Knot Theory to understand surfaces around us
- Did you know that a Sphere is the same as a Cube?!



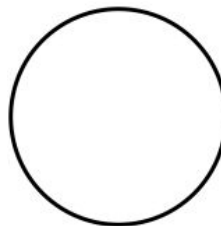
Brief Knot Theory Intro

- Knot Theory: Branch of Topology
 - Study of Mathematical Knots
- Mathematical knot: shoelace analogy!

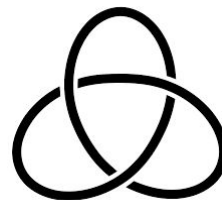


Examples of Mathematical Knots

Unknot



Trefoil Knot

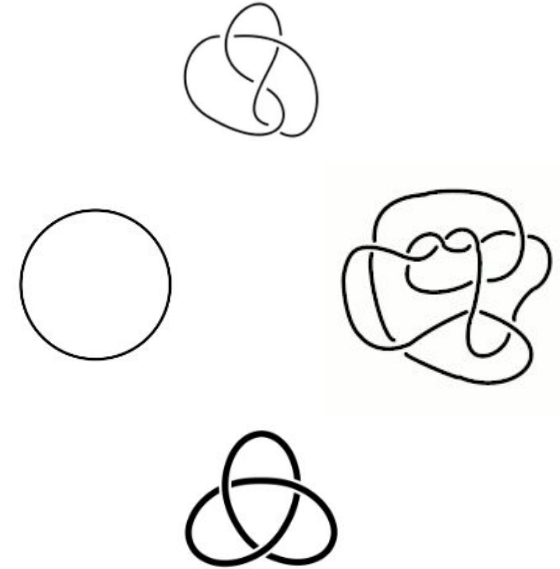


Knots: Theory and Terms

Knot: Infinitely thin line that has its two ends connected

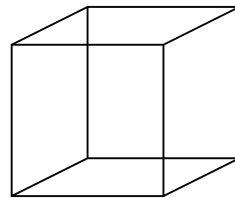
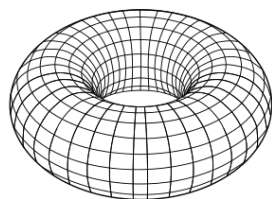
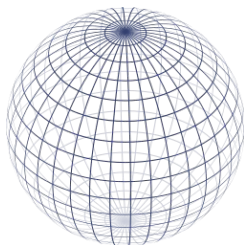
Unknot: Practically a circle. It is a knot with no crossings in it, the most basic form of a knot

Trefoil Knot: One of the most common knots, one with the lowest number of crossings

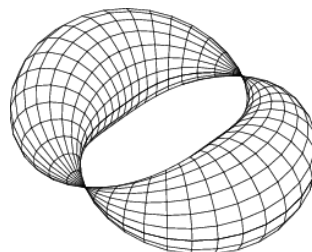
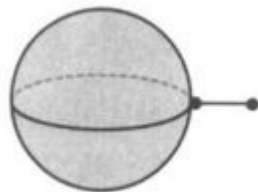
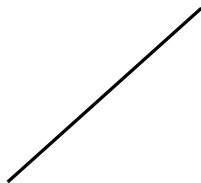


Surfaces and Non-Surfaces

Surface: Outside-most part of a three dimensional object

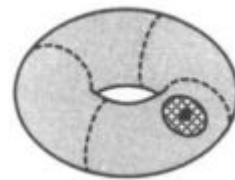


Examples of a **non surfaces**: Line or a Torus that is pinched infinitely thin

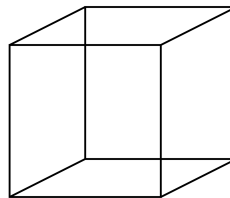
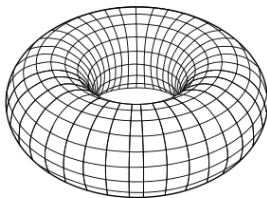
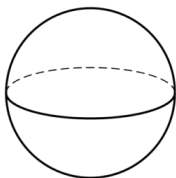


Surfaces Without Boundary

A surface without boundary is just a regular surface.



Examples of a surface without boundary: Sphere, Torus, and Cube



Surfaces With Boundary

A simple way of creating a boundary in a surface is by cutting a disk in it. Now the boundary of the surface is the empty space created by the disk.

Boundary Component: the missing circle.

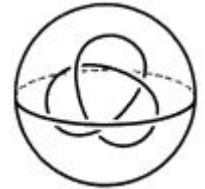
Example of a Surface with boundary:



Surfaces Relating To Knot Theory

- Every knot can be encapsulated in a torus
 - Just take an infinitely thin knot and make a tube around it

- Knots can even be encapsulated in a Sphere
 - The sphere is a complement of a knot = everything but the knot!



Surfaces: Euler Characteristic

Euler Characteristic: A number that describes a three dimensional object's shape or structure regardless of how it is oriented or disfigured.

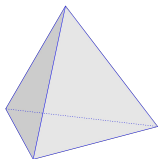


Leonhard Euler
(1707-1783)

Euler Characteristic = Vertices - Edges + Faces

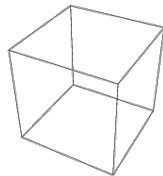
Tetrahedron

$$4 - 6 + 4 = 2$$



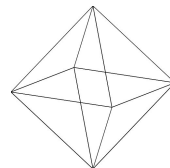
Cube

$$8 - 12 + 6 = 2$$



Octahedron

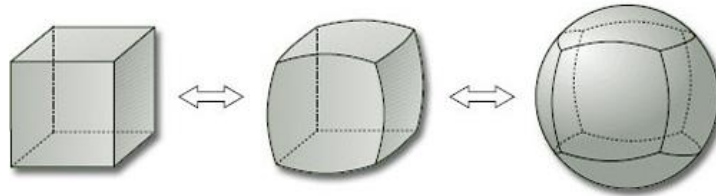
$$6 - 12 + 8 = 2$$



Surfaces: Isotopy

Isotopy: Two surfaces are *isotopic* when they are equivalent under rubber deformation

Look at this: a sphere and cube are mathematically the same!

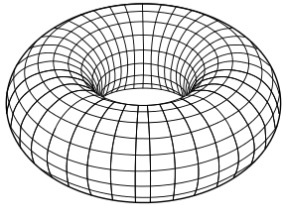


Surfaces: Genus

Genus: Number of through holes that an orientable surface has.

Examples of Orientable surfaces and their genera

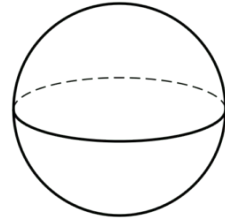
Torus: Genus = 1



Double torus: Genus = 2



Sphere : Genus = 0

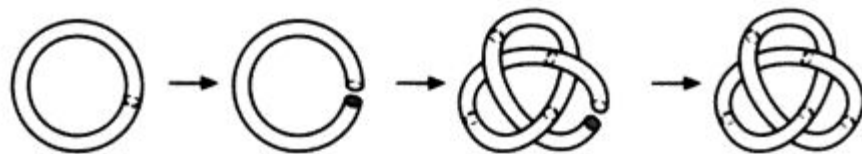


$$Euler\ Characteristic = 2 - 2 * Genus$$

Surfaces: Homeomorphism

Homeomorphic Surfaces: cut, change in any way, glue back exactly where it was connected before.

Unknot surface can be cut and weaved to look like a trefoil knot surface:



Homeomorphic does not mean “the same”!

Summary: What We Learned

- Knot Theory: fascinating subject in topology
 - A little hard to find resources on it ... but there are a lot of things you can do with it!
 - Can help in understanding of Biology, Chemistry and Physics
- Surfaces are found all around us
 - Some things are not intuitive: a Sphere and a Cube are the same thing!

Thank You

Mentor Kenneth Kox

Primes Circle

MIT

Parents

Thank You!

Questions?

Sources

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Wikipedia. en.wikipedia.org/wiki/Euler_characteristic.

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