On the Distortion of Torus Knots

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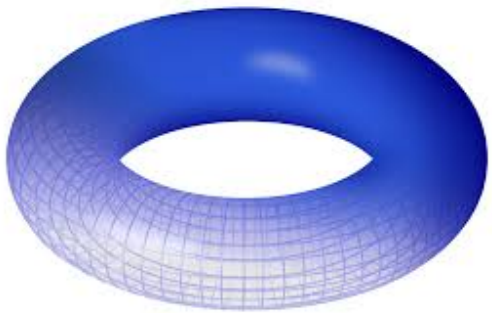
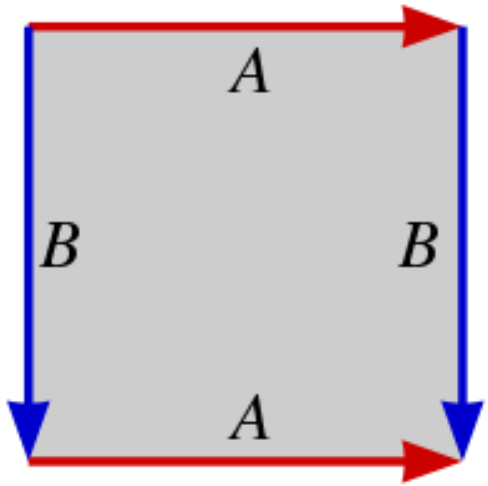
The Torus

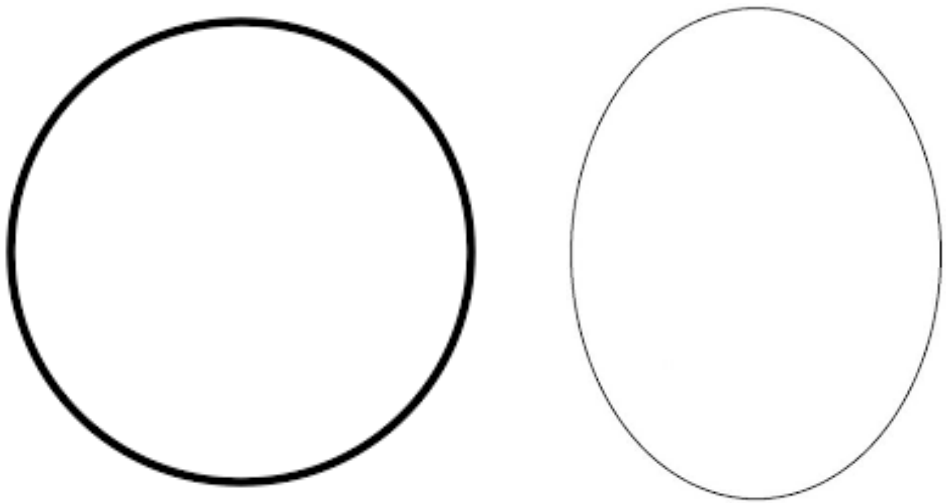
- The torus is an object in 3 dimensions which has a donut shape.
- It can also be thought of as a square with the left/right edges and top/bottom edges connected.





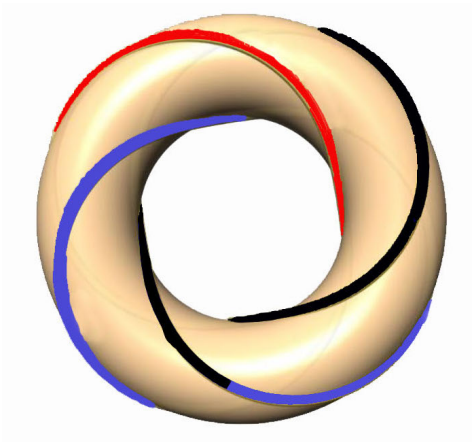
Knots in $\mathbb{R}^3$

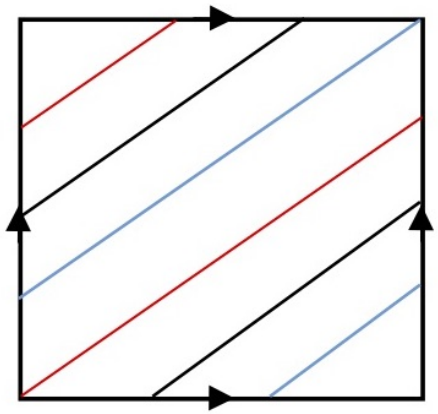
In $\mathbb{R}^3$, a knot K is an isotopy class of closed curves.



Knots on the Torus

- A torus knot is a closed loop placed on the surface of the torus.
  - It is completely determined by how many times it goes around the torus, akin to going across the edges of the square.
- The torus knot which goes around the outer circle p times and the inner circle q times is the p, q torus knot, written $T_{p,q}$.





The Distortion of a Curve

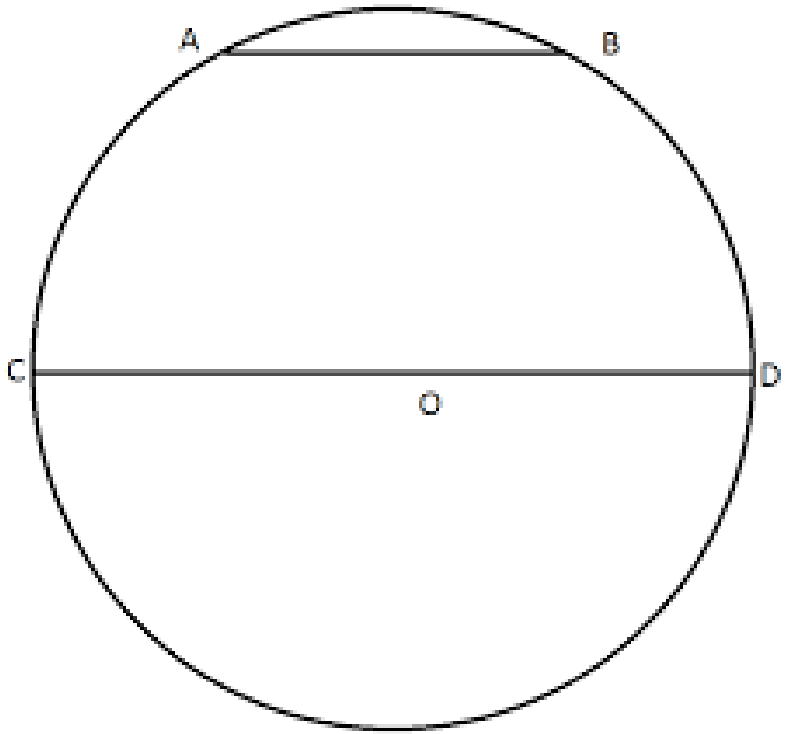
Let S be a curve in $\mathbb{R}^n$. The *distortion* of S, written $\delta(S)$, is defined as

$$\sup_{u,v \in S} \left(\frac{d_S(u,v)}{|u-v|} \right)$$

where $d(u,v)$ is the distance along S from u to v, and $|u-v|$ is the Euclidean distance.

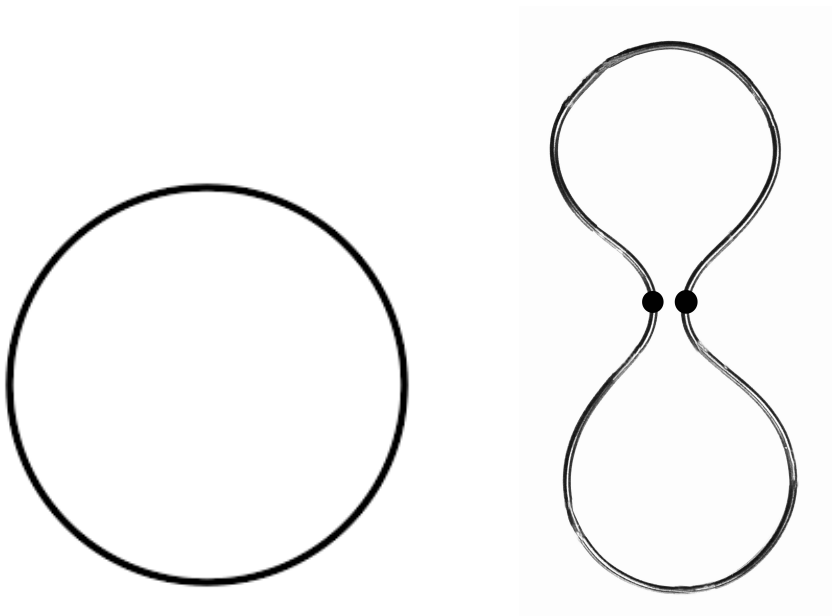
Distortion Example

The distortion of a circle is $\frac{\pi}{2}$.



Distortion of a Knot

Let K be a knot. There are many curves in $\mathbb{R}^3$ which may represent K, all isotopic to each other. The distortion of K, then, is the smallest distortion among all these curves, written $\delta(K)$.



Question, Gromov 1983

Question

Does every knot have a representative γ with $\delta(\gamma) < 100$? In particular, does such a γ exist for all torus knots $T_{p,q}$ as $p, q \to \infty$?

Theorem, Pardon 2010

Theorem

Consider the torus knot $T_{p,q}$. *Then,*
$\delta(T_{p,q}) > \frac{1}{160}\min(p,q)$.

Theorem, Studer 2015

Theorem

If $q \geq 50$, *then* $\delta(T_{2,q}) < \frac{7q}{\log(q)}$.

Our Main Result

Theorem

Let $q >> p$. *Then,* $\delta(T_{p,q}) < \frac{7q}{\log(q)}$.

Future Work

Conjecture

As $q \to \infty$, *we have* $\delta(T_{p,q}) \leq \frac{\pi(p-1)q}{p \log q}$.

Question

Can we extend this bound to the (p,q)-cablings of certain knots?

Question

How do we define the notion of average distortion, and how does it differ from Gromov's distortion?

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