

Approximating the dimension of circle packings

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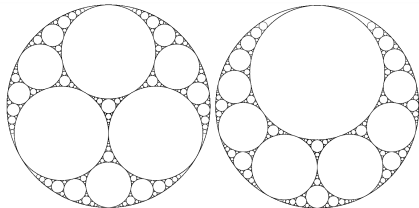
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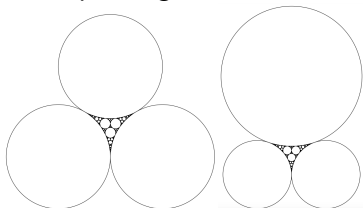
PRIMES-MIT Conference, May 18-19 2019

Round Packings

Apollonian Gaskets:



Other packings:



Conjecture (Chris Bishop):

The Apollonian Gasket has the lowest Hausdorff dimension of all round packings.

Fractional dimension

Box-counting dimension (Minkowski–Bouligand):



$$\dim_{\text{box}}(S) := \lim_{\varepsilon \rightarrow 0} \frac{\log N(\varepsilon)}{\log(1/\varepsilon)}$$

- Upper box dimension
- Lower box dimension

Fractional dimension (cont.)

Hausdorff measure:

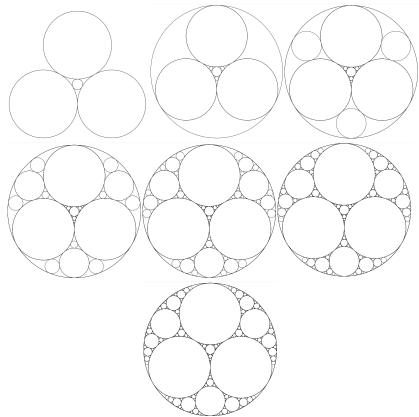
$$H^d(S) = \liminf_{\delta \rightarrow 0} \left\{ \sum_{i=1}^{\infty} (\text{diam } U_i)^d : \bigcup_{i=1}^{\infty} U_i \supseteq S, \text{diam } U_i < \delta \right\}$$

Hausdorff dimension:

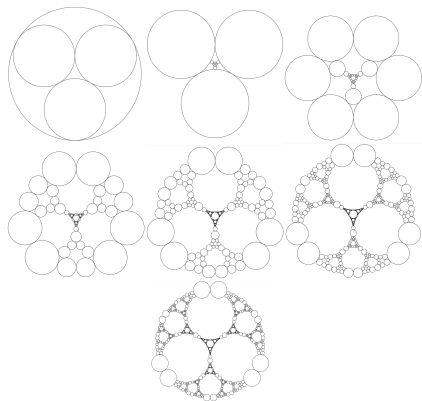
$$\dim_{\text{H}}(S) := \inf\{d \geq 0 : H^d(S) = 0\}$$

Markov Partition of Gasket

Gaskets:



Markov Partitions:



Current Results

Approximations of the Hausdorff dimension of gasket:

- 1.23965282
- 2.63825824
- 1.35901429
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- 1.28941077
- 1.30251193
- 1.2905869
- 1.29864229
- 1.29438865
- 1.29916915
- 1.29713521
- 1.30015633
- 1.29900686
- 1.30103601

McMullen: 1.305688

Boyd: Between 1.300197 and 1.314534

- Implementing further practical considerations
- Approximating dimensions of other gaskets
- Looking for patterns

Acknowledgments

I would like to thank my mentor, Professor Sergiy Merenkov, along with the PRIMES-USA program and its director, Doctor Slava Gerovitch. I would also like to thank the Head Mentor Doctor Tanya Khovanova, and the MIT math department.

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4. https://en.wikipedia.org/wiki/Minkowski–Bouligand_dimension