Reagan Choi

Stochastic Processes in Quantum Error Correction

under the direction of Andrey Boris Khesin

Abstract

We analyze surface codes in quantum error correcting. In these codes, qubits are encoded with a grid of cells, which may be affected by error. These errors cannot be detected directly; rather, we check the stabilizers of the encoding, which correspond to edges on the grid. This allows us to find the loops that surround the errors. We analyze the behavior of various processes that correct the errors on these loops. The absolute zero process is the most stable, and we run simulations to determine that it can correct a square loop of error in an average time of $O(n^3)$. We prove an upper bound for the absolute zero process and prove that the average time complexity of an altered process is $\Theta(n^3)$. Then, we analyze probabilistic algorithms. The behavior shown by the simulation of the probabilistic model indicates that there is a critical probability, approximately 0.175, at which error cannot reliably be corrected. We also analyze the heat bath algorithm, which can introduce errors to the grid but stochastically corrects large errors as long as the temperature is sufficiently small.

Kevin Cong

On the Sizes of Furstenberg Sets in Finite Fields

under the direction of Alexander Ortiz

Abstract

A (k,m)-*Furstenberg set* in $\mathbb{F}_q^n$ is a set of points such that every k-dimensional subspace of $\mathbb{F}_q^n$ has a translation containing at least m points of the set. We explore the question of estimating the size of the smallest (k,m)-Furstenberg set in $\mathbb{F}_q^n$, denoted by $K(q,n,k,m)$. We provide several general constructions for small Furstenberg sets which yield upper bounds on $K(q,n,k,m)$. In particular, we show that there is a universal constant C such that for large m, $K(q,n,1,m) \leq Cq^{\frac{n-1}{2}}m^{\frac{n+1}{2}}$, which is not far from the known lower bounds. We show another upper bound that $K(p^l,n,1,p^{l-1}) \leq Cm^2$, constituting an optimal upper bound up to constants. We also generalize existing lower bounds when $k = \sqrt{q}$, constituting an improvement of the easy lower bound Cm^2. Finally, we suggest other methods to potentially obtain new lower bounds.

Allen Lin

On the Properties of Polyas Circular Symmetrization

under the direction of Sarah Tammen

Abstract

The isoperimetric inequality of a region $\Omega \subset \mathbb{R}^n$ is an inequality comparing $\mathcal{L}^{n-1}(\partial\Omega)$ to $\mathcal{L}^n(\Omega)$. Many symmetrization methods have been developed to prove this, including Steiner $S_L(\Omega)$ and circular $\text{Circ}(\Omega)$ symmetrizations. The underlying idea in these symmetrizations is that they preserve volume but decrease perimeter or surface area. This can be easily shown using calculus. We present an elementary, non-calculus, proof that circular symmetrization decreases the perimeter of a region in $\mathbb{R}^2$. We then show that the diameter of a region entirely to the right of the y-axis decreases after circular symmetrization if the intersection of the region with an arc of radius r is either a single arc or empty for all $r > 0$. Finally, we present different regions Ω and their respective circular symmetrizations $\text{Circ}(\Omega)$.

On Ki Luo

Investigation on the Johnson-Leader-Russell Question for Square Posets

under the direction of Pakawut Jiradilok

Abstract

We study a problem proposed by Johnson, Leader, and Russell. Given positive integers n and k, we aim to find the maximum number of maximal chains in a subset with size k of a square poset $\mathfrak{P} = \{1, 2, ..., n\}^2$. Kittipassorn made progress on this problem by solving a stronger case of which the number of elements in each level is also given. With his work, we find the exact solution for $1 \leq k \leq 3n-2$. For general k, we find that the optimal configuration is given by a 1-Lipschitz function, i.e. the difference between the number of elements in two consecutive levels is at most 1.

Eli Meyers

Homological Equivalence and Forman Equivalence of Discrete Morse Functions on Graphs

under the direction of Mary Stelow

Abstract

Discrete Morse theory is a relatively recent technique which has proven to be a useful tool in diverse areas such as topology, computer science, and data analysis/denoising. Due to its discrete formulation, it is possible to study the set of all discrete Morse functions on a simplicial complex up to different notions of equivalence. The notion of homological equivalence was introduced by Ayala et. al, who obtained a count for the number of homological equivalence classes of discrete Morse functions on a given finite graph. The relationship between homological and Forman equivalence for discrete Morse functions on trees was described recently. In our paper, we generalize their study of the relationship between these notions of equivalence to all graphs. To do so, we develop the concepts of critical graphs and critical matrices, which describe the CW decomposition of a graph given by a specific discrete Morse function. We use these results to obtain a new proof of Ayala et al.'s characterization of possible homological equivalence classes of Morse functions on a given graph.

Comfort Ohajunwa

On Phase Transitions in the Approximation Ratio for MAX 2-SAT

under the direction of Mitchell Harris

Abstract

In the Boolean Maximum 2-Satisfiability (MAX 2-SAT) problem, we consider a Boolean formula $\mathcal{F}$ in conjunctive normal form with clauses that contain only two literals each, and ask for the maximum number of clauses that can be satisfied in $\mathcal{F}$. MAX 2-SAT is an NP-complete problem, and it has been identified that it undergoes a phase transition where the hardness shifts from underconstrained to overconstrained. The hardness of MAX 2-SAT depends on the clause to variable ratio c, such that MAX 2-SAT undergoes a phase transition when $c = 1$. Meanwhile, there exist approximation algorithms to estimate solutions to MAX 2-SAT in polynomial time using relaxations. One such relaxation involves the use of semidefinite programs (SDP), in which the algorithm optimizes over the cone of positive semidefinite matrices, or matrices with nonnegative eigenvalues. Semidefinite programming was first incorporated into an approximation algorithm for MAX 2-SAT by Goemans and Williamson, which produces approximations such that the exact solution is at least 0.87856 times the SDP upper bound. Yet, despite our knowledge of MAX 2-SAT's behavior over different regions of constrainedness, less can be said about how the ratio between the exact solution and the approximated solution, or the approximation ratio, changes over these regions. Thus, we studied how the approximation ratio changes over c to identify if there exists a phase transition for the approximation ratio. Ultimately, we found that the approximation ratio approaches 1 as MAX 2-SAT becomes more constrained. In addition, when the SDP upper bound is restricted such that it cannot produce solutions greater than the number of clauses in an instance, there is an easy-hard-easy pattern for the approximation ratio. We also found that when the negation of variables in MAX 2-SAT instances are "unbalanced," such that variables are negated with a probability of 32%, the rate of convergence between the exact solution and upper bound is faster and the approximation ratio is greater for larger c.

Isabella Quan

On Snowflakes and Pizza: Graph Theoretic Properties of 2D Steiner Solutions

under the direction of Mary Stelow

Abstract

Steiner's plane cutting problem asks for configurations of lines in the plane that create the maximum number of regions (termed S-solutions). Following previous work, we associate to every S-solution a graph in the plane and establish graph theoretic properties common to these graphs. We also introduce a new technique for analyzing and constructing S-solutions, called the "snowflake transformation". This allows us to construct families of S_n-solutions where we can bound the maximum vertex degree between 4 and n, construct traceable diagrams with greater ease, and have large maximum independent sets. In future research, we want to generalize this to n dimensions, as well as find necessary conditions for a given graph to be an S-solution.

Angel Raychev

A Generalization of Descent Polynomials

under the direction of Pakawut Jiradilok

Abstract

The notion of a *descent polynomial*, a function in enumerative combinatorics, that counts permutations with specific properties, enjoys a revived recent research interest due to its connection with other important notions in combinatorics, viz. *peak polynomials* and *symmetric functions*. We define the function $\mathfrak{d}^m(I, n)$ as a generalization of the descent polynomial (for which $m = 1$) and we prove that this function is a polynomial in n for sufficiently large n (similarly to the descent polynomial). We look at the coefficients of $\mathfrak{d}^m(I, n)$ in falling factorial bases. We prove positivity of the coefficients and discover a combinatorial interpretation for them. This result generalizes the positivity result of Diaz-Lopez et al. for the descent polynomial. We obtain an explicit formula for $\mathfrak{d}^m(I, n)$ in some special cases.

Tair Satubaldin

Examples for the Robust Qualitative Uncertainty Principle

under the direction of Alexander Ortiz

Abstract

In this paper, we explore the sharpness of the qualitative robust uncertainty principle by inspecting the examples of different functions that could possibly extremize the uncertainty principle. We look at the examples of one Gaussian, sum of Gaussians arranged in a finite arithmetic progression and the blurred and truncated Dirac comb. For each example, we approximate the product of measures of two smallest sets, where the function and its Fourier transform are $\frac{1}{10}$-concentrated. In the course of this research, we get that in all listed examples, the value of the explored product is approximately the same as in case of a Gaussian function.

Daniel Xia

An Inequality for the Antiferromagnetic Potts Model

under the direction of David Jongwon Lee

Abstract

In this paper, we study a graph-theoretic inequality on a general Potts model partition function $Z(G, \Omega_V)$, allowing weights on the vertices and colors. We first discuss a purely algebraic inequality, which we prove for integral and sufficiently large real exponents. From this algebraic inequality, we deduce several local graph-theoretic inequalities. For the case $\beta = 0$, which counts the number of weighted proper colorings, we use these local inequalities to prove, by induction, an upper bound on $Z(G, \Omega_V)$ in terms of bipartite graphs.

Lucy Xiao

The Adaptive Capacity for Two Mixed States

under the direction of Andrey Boris Khesin

Abstract

In this paper, we investigate the one-shot capacity $C_{1,1}$, and the adaptive one-shot capacity $C_{1,A}$ in transmitting classical information through quantum channels. Capacity $C_{1,A}$ allows for an adaptive strategy where the receiver can make non-disruptive measurements and adapt using the results of previous measurements. Shor proves that $C_{1,1} = C_{1,A}$ for two pure states, and conjectures that the same is true for mixed states. We show that the accessible information is concave in the probability of the states, then conclude $C_{1,1} = C_{1,A}$ for any two mixed states in arbitrary dimensions, proving the conjecture.