Graph Theory and Its Applications

By: Crystal Egbunike and Wintana Tewolde

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But first… why might we care about Graph Theory?

Preview of Important Applications of Graph Theory

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- **Dijkstra Algorithm** is used to help find the shortest path from one point to another.
- **PageRank** is used to determine the most relevant webpages when a user makes a search in a search engine.
- Both of these applications are used by us everyday without even realizing it.

What is a Graph?

Graph: a collection of edges and vertices

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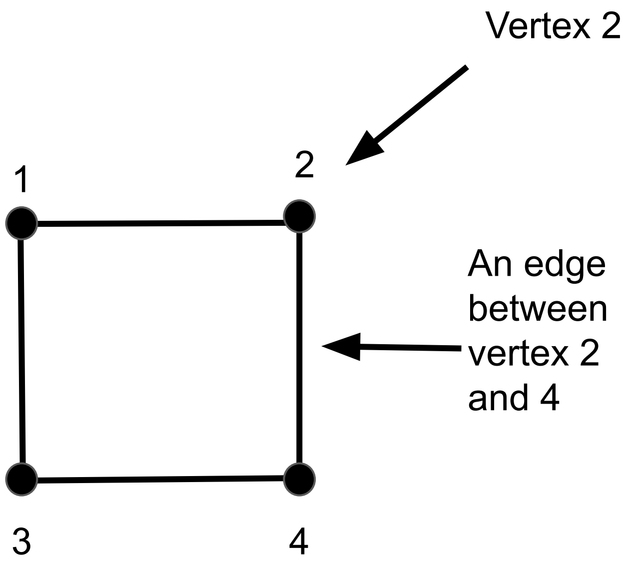

An example of a graph with a labelled edge and vertex

What is a Matrix?

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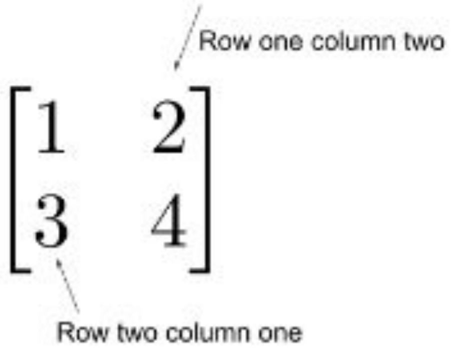

An example of a matrix with two labelled entries

Weighted Graph

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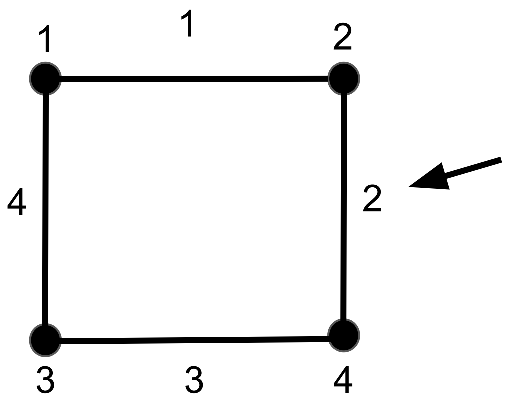

An example of a an edge between vertex 2 and 4 with a weight of 2.

An example of a weighted graph with a labelled weighted edge

Directed Graph

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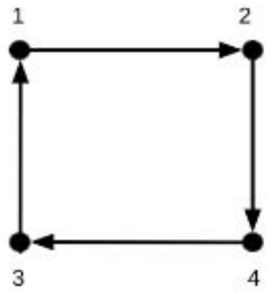

An example of a directed edge leaving vertex 2 and arriving at vertex 4.

A directed graph with a labelled directed edge.

Incidence Matrix

Incidence Matrix:

- The matrix of a directed graph made up of 1’s, -1’s, and 0’s
  - 1 = leaving a vertex
  - -1 = arriving at a vertex
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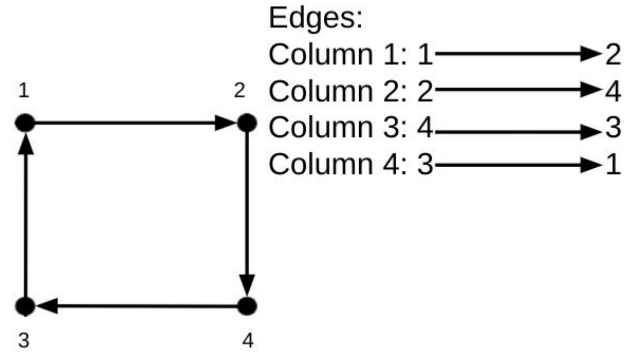

A directed graph with each edge assigned to a particular column in the incidence matrix.

This is an example of an incidence matrix created based on a directed graph.

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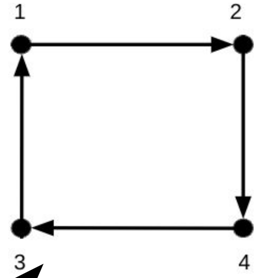


Edges:
Column 1: 1 → 2
Column 2: 2 → 4
Column 3: 4 → 3
Column 4: 3 → 1

A directed graph with each edge assigned to a particular column in the incidence matrix.

An incidence matrix to represent the graph above.

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

This entry of -1 represents that the edge between vertex 3 and 1 is leaving vertex 3 and arriving at vertex 1.

This is an example of an incidence matrix created based on a directed graph.

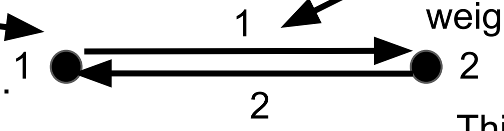
Transition Matrix

Transition Matrix:

- A matrix based on the weights assigned to each edge in a directed weighted graph
- Rows represent the weight of the edges arriving at a vertex.
- Columns represent the weight of the edges departing from a vertex.

A simple directed weighted graph.

An edge between vertex 1 and 2 with a weight of 1.



This entry of 2 represents the weight of the edge arriving at vertex 1 and leaving vertex 2.

A transition matrix based on the graph above.

$$\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$$

This is an example of a transition matrix created based on a directed weighted graph.

So how can all of this information be applied?

1st Application of Graph Theory: Dijkstra Algorithm

- The **Dijkstra Algorithm** is used to find the shortest path from the ‘source node/vertex’ to all the adjacent/surrounding nodes.

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- This method is still currently used by navigational apps, including Google Maps, Apple Maps, and more.

Google Maps

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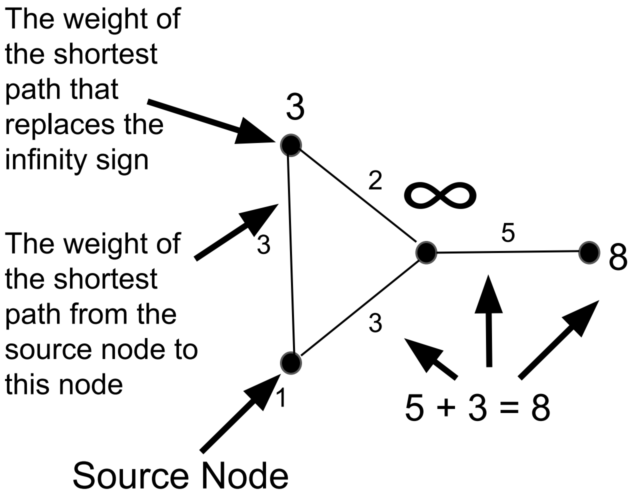

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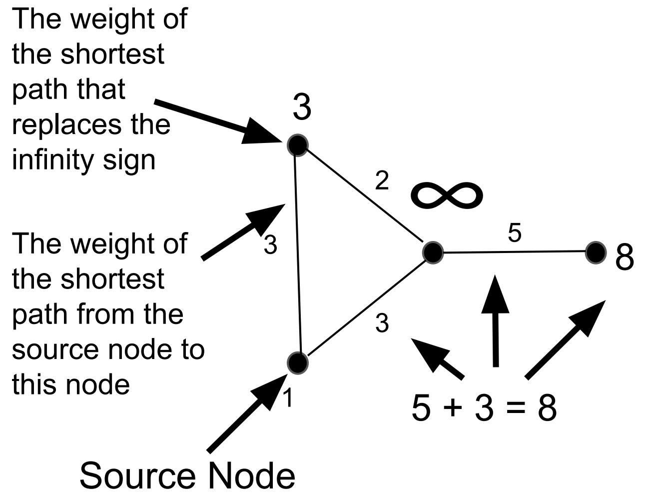

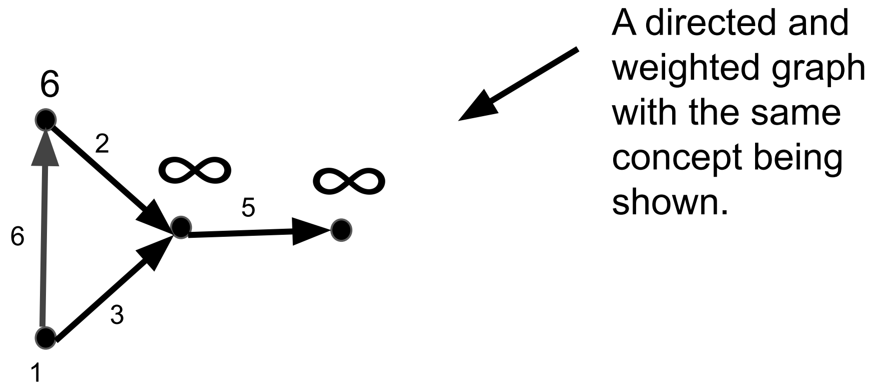

2nd Application of Graph Theory: PageRank System

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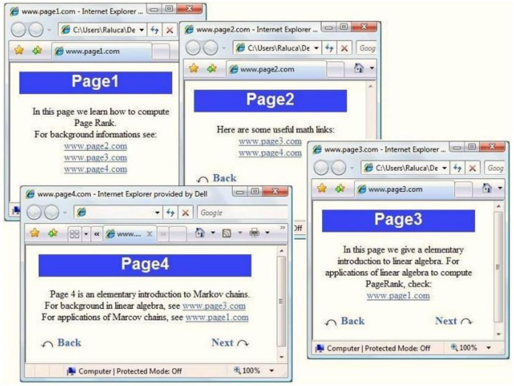

An example of what page ranking may look like. Because all the other web pages link to webpage 3, webpage 3 would be considered the most important.

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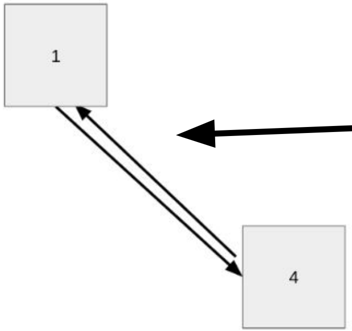
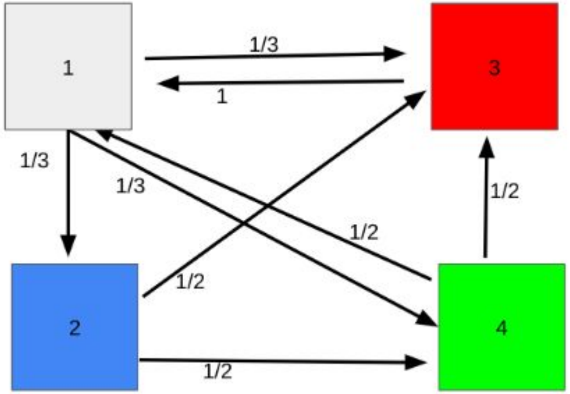


This is an example of the connections between website 1 and 4. The graph shows that website 1 links to 4 and website 4 links to website 1.

Page Rank Can Also Be Represented by a Directed Weighted Graph and Transition Matrix as Shown Below:



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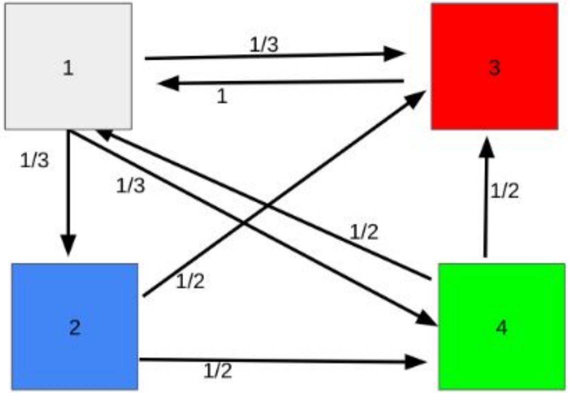

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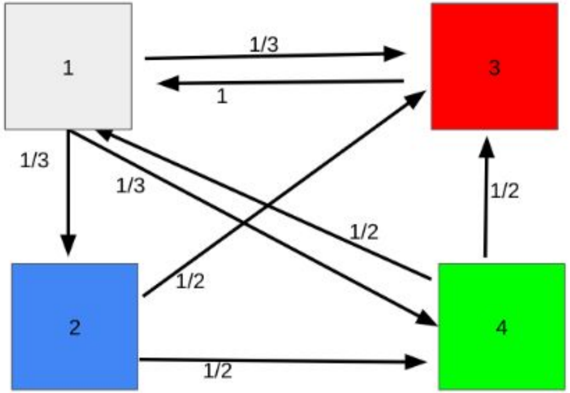

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This is a transition matrix created based on the graph to the left.

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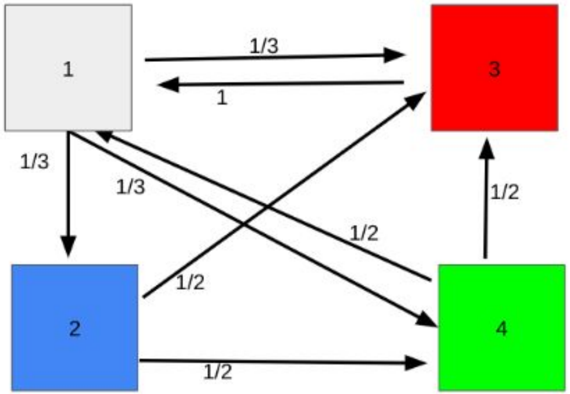

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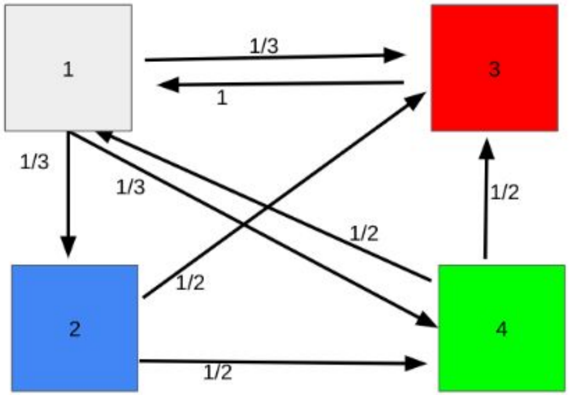

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Thanks for Listening!