Motion Analysis via Topological Tools

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Data Size

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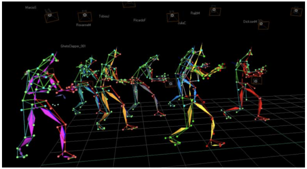

While straightforward, these methods limit motion capture to the confines of well-funded businesses.

Increasing Demands

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Effective in revealing flaws or weaknesses in a player's or a dancer's technique.

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Performance arts: gives a scientific evaluation of what makes certain movements aesthetically appealing.

Network Topology

Consider the subject of motion capture as graph G.
We can define the graph as follows:

Definition

$G = (V, E)$ where V stands for *vertices* and E stands for edges.

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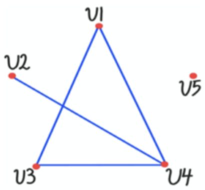

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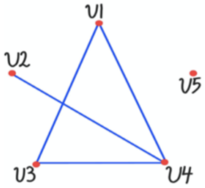

1. V (vertices): $\{v_1, v_2, v_3, v_4, v_5\}$
2. E (edges): $\{(v_1, v_3), (v_1, v_4), (v_2, v_4), (v_3, v_4)\}$

Matrix

In this case, we can express these sets as the following matrix.

$$N(V,E) = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$V = \{v_1, v_2, v_3, v_4, v_5\},\ E = \{(v_1, v_3), (v_1, v_4), (v_2, v_4), (v_3, v_4)\}$$

With the given matrix, we can then further compress the data by defining the topology of the given graph (G, E).

Grouping Movements

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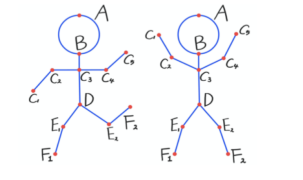
Example



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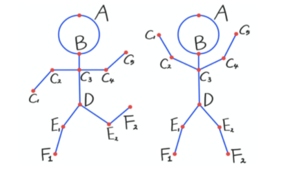

$$N(V,E) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

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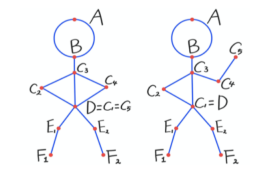

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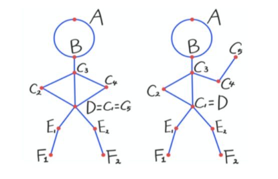

$$N(V,E)=\begin{bmatrix}
0&1&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&1&0&0&1&1&0&0&0&0\\
0&0&1&0&1&0&0&0&0&0&0&0\\
0&1&0&1&0&1&0&1&0&0&0&0\\
0&0&0&0&1&0&1&0&0&0&0&0\\
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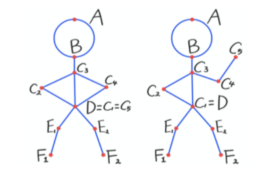

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Topological Manifolds: Adding rotations

In a general sense, a topological manifold is a topological space that locally resembles a real n-dimensional Euclidean space.

For any point in the space, there exists an open set containing the point, and we can create a *smooth map* (infinitely differentiable and invertible) between the open set of the given topological manifold and some open set of the Euclidean space.

The Orthogonal Group

The definition of an orthogonal group is given as follows:

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Definition

$$O(n) = \{\mathbb{R} \in M_{n\times n} : \mathbb{R}\mathbb{R}^\top = I_n\},$$

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Then, assuming that one of the two body parts is fixed in position, we can say that the other body part's position can be corresponded to an element of $O(3)$.

Example of Topological Analysis in Modern Dance

Example

Lewitzky's motion at 0:27

$$N_L(V,E) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

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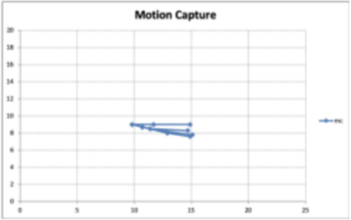

Example of Topological Analysis in Modern Dance

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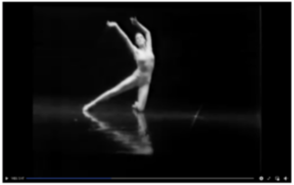

Lewitzky's motion at 1:02

$$N_R(V,E) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

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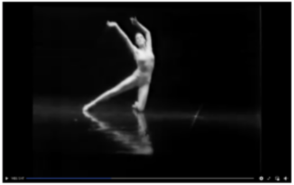

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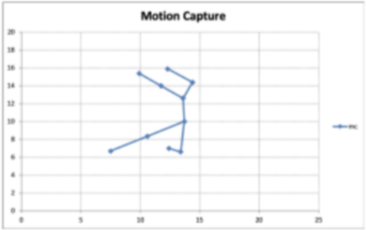

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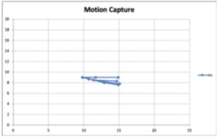
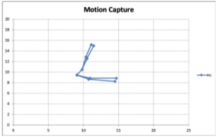
Lewitzky's motion at 0:27

Lewitzky's motion at 0:32

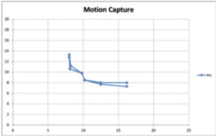
Lewitzky's motion at 0:36



Lewitzky's motion capture via network topology at 0:27



Lewitzky's motion capture via network topology at 0:32



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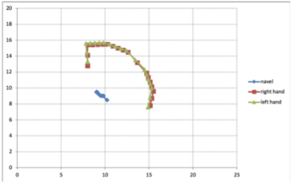
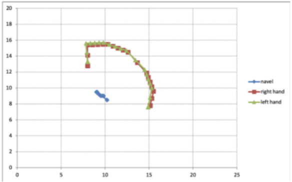


The path of Lewitzky's hand and navel from 0:27 - 0:36, Method 1

Example of Topological Analysis in Modern Dance



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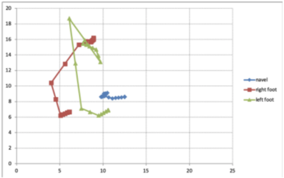
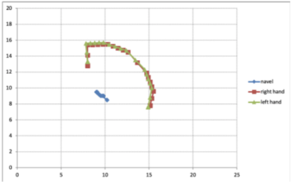


The path of Lewitzky's feet and navel from 0:44 - 0:52

Example of Topological Analysis in Modern Dance



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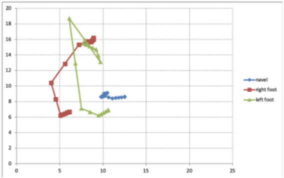

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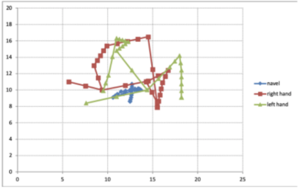

The path of Lewitzky's hand and navel from 0:52 - 1:06

Summary

1. **Network Topology**:
   Considers the human body as a graph. Instead of plotting specific points all around, we simply connect two vertices with a vector line.
2. **Topological Manifolds**:
   Investigates the states of joints as an element of $O(3)$ to express the human body.
   (More research needed: calculating the rotations required more data than given by the 20th century video file (torsion, for example).)

Click the link to access the full paper:
https://tinyurl.com/2hejdzzc

THANK YOU!

References

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