# Applications of a Modified Age-Structured SIR Model

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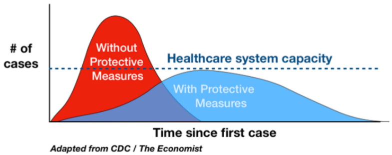
# Importance and Motivation

Models provide a useful tool for planning and decision making.

- COVID-19 Pandemic
- Predicting Spread
- Understanding effect of
  - Social-distancing measures
    - Opening schools?
  - Stress on healthcare systems
    - ICU Capacity
  - Vaccine Distribution



# SIR (Susceptible, Infected, Removed)

The number of individuals in each compartment is given by certain functions of time $S(t)$, $I(t)$, $R(t)$ respectively. $(\frac{\mathrm{d}S}{\mathrm{d}t} + \frac{\mathrm{d}I}{\mathrm{d}t} + \frac{\mathrm{d}R}{\mathrm{d}t} = 0)$

## SIR Model

$$\begin{aligned}
\frac{\mathrm{d}S}{\mathrm{d}t} &= -\beta \cdot I \cdot \frac{S}{N} \\
\frac{\mathrm{d}I}{\mathrm{d}t} &= \beta \cdot I \cdot \frac{S}{N} - \gamma \cdot I \\
\frac{\mathrm{d}R}{\mathrm{d}t} &= \gamma \cdot I
\end{aligned}$$

Population size $N$, transmission rate $\beta$, removal rate $\gamma = 1/D$ where $D$ is recovery time.

# SIR Dynamics


Figure: Transfer of individuals between compartments

# Adding Age Specificity

Vector valued functions $S(t), I(t), R(t)$ with $X_i$ denoting the number of individuals in compartment $X$ in the $i$th age-bracket.

## Age-Specific SIR Model

$$\begin{aligned} \frac{\mathrm{d}S_i}{\mathrm{d}t} &= -\beta \cdot \frac{S_i}{N} \cdot \sum_{j=1}^{n} \mathcal{M}_{ij} \cdot I_j \\ \frac{\mathrm{d}I_i}{\mathrm{d}t} &= \beta \cdot \frac{S_i}{N} \cdot \sum_{j=1}^{n} \mathcal{M}_{ij} \cdot I_j - \gamma \cdot I_i \\ \frac{\mathrm{d}R_i}{\mathrm{d}t} &= \gamma \cdot I_i \end{aligned}$$

We can also compute the number of individuals hospitalized, in critical care, and deceased: $\frac{\mathrm{d}H_i}{\mathrm{d}t} = \gamma \cdot h_i \cdot I_i$, $\frac{\mathrm{d}C_i}{\mathrm{d}t} = \gamma \cdot h_i \cdot c_i \cdot I_i$, $\frac{\mathrm{d}M_i}{\mathrm{d}t} = \gamma \cdot m_i \cdot I_i$.

# Contact Matrix and Reproductive Number

- Basic reproduction number ($R_0$) is the initial expected growth rate in a fully susceptible population.
- $R_0 = 5.7$ for COVID-19.
- If $\lambda$ and $\boldsymbol{v}$ are the dominant eigenvalue and eigenvector of $\mathcal{M} \cdot \mathrm{diag}(\boldsymbol{p})$.
  - $\beta = \frac{R_0 \cdot \gamma}{\lambda}$.
  - Initial $\boldsymbol{I} = \boldsymbol{v} \cdot I_{\text{tot}}$.

| Age | 0-9 | 10-19 | 20-29 | 30-39 | 40-49 | 50-59 | 60-69 | 70-79 | 80+ |
|---|---|---|---|---|---|---|---|---|---|
| 0-9 | 19.2 | 4.8 | 3.0 | 7.1 | 3.7 | 3.1 | 2.3 | 1.4 | 1.4 |
| 10-19 | 4.8 | 42.4 | 6.4 | 5.4 | 7.5 | 5.0 | 1.8 | 1.7 | 1.7 |
| 20-29 | 3.0 | 6.4 | 20.7 | 9.2 | 7.1 | 6.3 | 2.0 | 0.9 | 0.9 |
| 30-39 | 7.1 | 5.4 | 9.2 | 16.9 | 10.1 | 6.8 | 3.4 | 1.5 | 1.5 |
| 40-49 | 3.7 | 7.5 | 7.1 | 10.1 | 13.1 | 7.4 | 2.6 | 2.1 | 2.1 |
| 50-59 | 3.1 | 5.0 | 6.3 | 6.8 | 7.4 | 10.4 | 3.5 | 1.8 | 1.8 |
| 60-69 | 2.3 | 1.8 | 2.0 | 3.4 | 2.6 | 3.5 | 7.5 | 3.2 | 3.2 |
| 70-79 | 1.4 | 1.7 | 0.9 | 1.5 | 2.1 | 1.8 | 3.2 | 7.2 | 7.2 |
| 80+ | 1.4 | 1.7 | 0.9 | 1.5 | 2.1 | 1.8 | 3.2 | 7.2 | 7.2 |

Figure: Contact Matrix $\mathcal{M}$

# Social-distancing policy in Washington State

Since late May, the Safe Start policy has been implemented in Washington:

- Phased reopening plan that gradually relaxes social distancing measures based on assessments of health care system readiness.
- We model health care system stress by considering the proportion of occupied ICU beds (based on an average of 3.47 beds per 100,000 residents).

$$\mathcal{M} \to \frac{1}{\lambda \cdot |C|/C_{\mathrm{max}}} \mathcal{M}.$$

- We denote $\frac{1}{\lambda \cdot |C|/C_{\mathrm{max}}}$ as the mitigation-factor and select $\lambda = 0.2$ based on real infection points in infection count.

# Population Parameters

We select four counties (Jefferson, King, Ferry, Adams with median ages 28, 36, 49, 58 respectively) and apply our model to their demographic parameters.

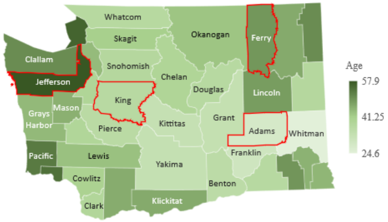



Figure: County Median Age

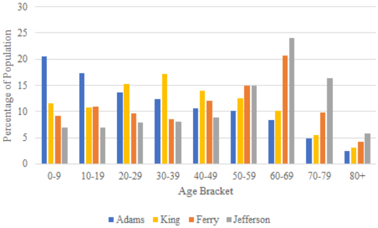



Figure: Age Distribution

# SIR Application

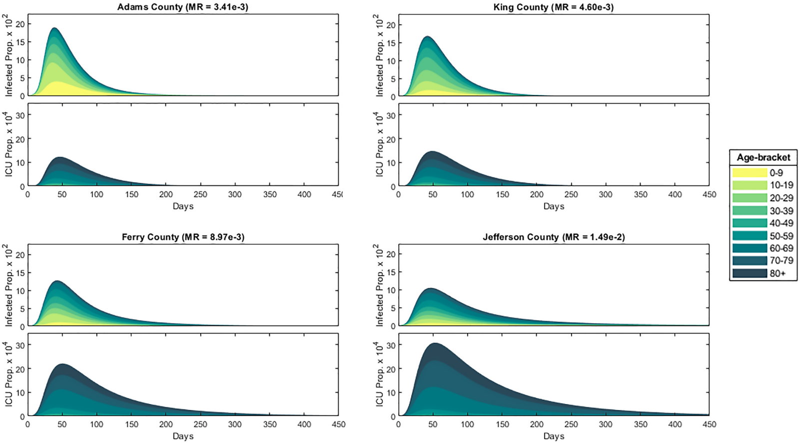



Figure: County Infected and ICU Proportion (Homogeneous Mitigation)

# SIR Results

- As median age increased, peak infections decreased while peak ICU occupancy and mortality rate increased.
- For all counties, approximately the same proportion (80%) of the population were infected.
- Herd immunity threshold $1 - 1/R_0 \approx 82\%$.

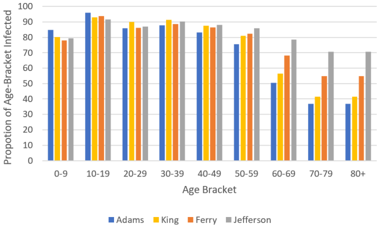



Figure: Proportion of Age-Bracket Infected

# Age-Targeted Relaxation

We examine the effects of relaxing restrictions for two age groups:

- The school bracket: 0-29 years
- The work bracket: 0-69 years

We also consider relaxing at the start of the epidemic (at 0 days) and after the initial peak in infections (at 90 days).

| Age | 0-9 | 10-19 | 20-29 | 30-39 | 40-49 | 50-59 | 60-69 | 70-79 | 80+ |
|---|---|---|---|---|---|---|---|---|---|
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| 70-79 | 1.4 | 1.7 | 0.9 | 1.5 | 2.1 | 1.8 | 3.2 | 7.2 | 7.2 |
| 80+ | 1.4 | 1.7 | 0.9 | 1.5 | 2.1 | 1.8 | 3.2 | 7.2 | 7.2 |

Figure: Blue group is not subject to the mitigation-factor when relaxing 0-39 bracket

# Age-Targeted Relaxation - Application

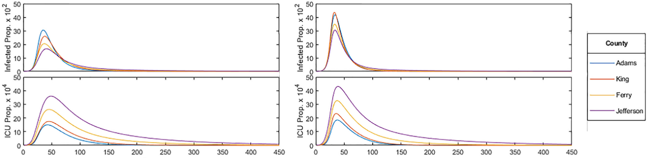


Figure: School and Work Bracket Relaxation at 0 days

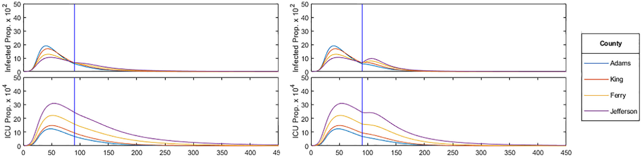


Figure: School and Work Bracket Relaxation at 90 days

# Age-Targeted Relaxation - Results

At 0 days:

- Peak infections increased (on average) by 58% when relaxing school bracket and increased by 160% when relaxing work bracket.
- Although the calculated mortality rate decreased slightly (by 6% and 12% respectively), excessive stress was placed on hospitals with a 51% increase in peak ICU occupancy when relaxing work bracket.
- Placing restrictions on the school and work brackets has the effect of "flattening the curve".

At 90 days:

- Relaxing school bracket had little effect on subsequent trajectory of epidemic with mortality rate decreasing by an average 3.5%.
- Relaxing work restrictions had a significant effect on high median age populations with Jefferson experiencing a new peak in infections and increasing ICU occupancy for over a month after 90 days.

# Age-Targeted Vaccination

We examine the efficacy of age-targeted vaccine distributions.

- Individual transferred directly from susceptible to removed compartments.
- A constant $(N/720)$ vaccines are administered to the population at each time step.
- Vaccines are distributed to each age-bracket according to a weight vector $\omega$.
  - $\omega_C$ constant (homogeneous) distribution
  - $\omega_M$ moderate priority
  - $\omega_S$ strict priority

| Age | $\omega_C$ | $\omega_M$ | $\omega_S$ |
|---|---|---|---|
| 0-9 | 1 | 1 | 1 |
| 10-19 | 1 | 1 | 1 |
| 20-29 | 1 | 1 | 1 |
| 30-39 | 1 | 1 | 1 |
| 40-49 | 1 | 1 | 2 |
| 50-59 | 1 | 2 | 4 |
| 60-69 | 1 | 2 | 8 |
| 70-79 | 1 | 4 | 16 |
| 80+ | 1 | 4 | 16 |

Figure: Distribution Weights

# Age-Targeted Vaccination - Application

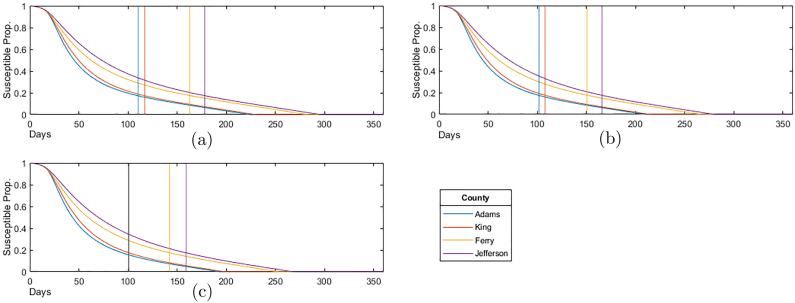


Figure: (a) Homogeneous (b) Moderate Priority (c) Strict Priority

# Age-Targeted Vaccination - Results

- Homogeneous vaccine distribution reduced mortality rate by an average 28.2% compared to without vaccines. Only 59.2% of Jefferson county became infected.
- Age-targeted distributions further reduced mortality rate, especially among low median age counties: by 9.4% and 19.5% in Adams county for $\omega_M$ and $\omega_S$ respectively.
  - Time required to achieve HIT decreased by 7.3% and 11.6% on average for $\omega_M$ and $\omega_S$ respectively.

| County | $\omega_C$ | $\omega_M$ | $\omega_S$ |
|---|---|---|---|
| Adams | 110 | 102 | 100 |
| King | 117 | 108 | 101 |
| Ferry | 163 | 151 | 142 |
| Jefferson | 178 | 166 | 159 |

Figure: Days required to achieve Herd Immunity Threshold

# Further Work

Some directions for potential further research:

- Model different population's social distancing patterns including restrictions based on hospitalization count or infection positive rate.
- Implement interactions between school and work populations as separate compartments.
- Consider movement of individuals between counties.
- Vaccine distribution that varies over time.

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# References


Acemoglu, D., Chernozhukov, V., Werning, I., and Whinston, M. D. (2020). Optimal targeted lockdowns in a multi-group sir model. *NBER Working Paper*, 27102.

Baker, R. (2020). Reactive social distancing in a sir model of epidemics such as covid-19.

Bloom, B. S., Hillman, A. L., Fendrick, A. M., and Schwartz, J. S. (1993). A reappraisal of hepatitis b virus vaccination strategies using cost-effectiveness analysis. *Annals of Internal Medicine*, 118(4):298–306.

Chikina, M. and Pegden, W. (2020). Modeling strict age-targeted mitigation strategies for covid-19.

Ferguson, N. et al. (2020). Report 9: Impact of non-pharmaceutical interventions (npis) to reduce covid-19 mortality and healthcare demand.

Fernandez, E. and Diamond, M. S. (2017). Vaccination strategies against zika virus. *Current opinion in virology*, 23:59–67.

for Disease Control, C. and Prevention (2020). https://www.cdc.gov/flu/fluvaxview/coverage-1718estimates.htm.

Guerra, F. M., Bolotin, S., Lim, G., Heffernan, J., Deeks, S. L., Li, Y., and Crowcroft, N. S. (2017). The basic reproduction number (r0) of measles: a systematic review. *The Lancet Infectious Diseases*, 17(12):e420–e428.

Keeling, M. J. and Eames, K. T. (2005). Networks and epidemic models. *Journal of the Royal Society Interface*, 2(4):295–307.

# References (cont.)

Kermack, W. O. and McKendrick, A. G. (1927). A contribution to the mathematical theory of epidemics. *Proceedings of the royal society of london. Series A, Containing papers of a mathematical and physical character*, 115(772):700–721.

Mandal, S., Sarkar, R. R., and Sinha, S. (2011). Mathematical models of malaria-a review. *Malaria journal*, 10(1):1–19.

of Financial Managment, O. (2020). https://www.ofm.wa.gov/washington-data-research/statewide-data/washington-trends/population-changes/population-age-mapped-county.

of Health, W. S. D. (2020). https://www.doh.wa.gov/emergencies/coronavirus.

Pastor-Satorras, R. and Vespignani, A. (2001). Epidemic dynamics and endemic states in complex networks. *Physical Review E*, 63(6):066117.

Prem, K. and Cook, A. R. (2017). Projecting social contact matrices in 152 countries using contact surveys and demographic data. *PLoS Comput Biol*, 13.

Sanche, S. and Lin, Y. T. (2020). High contagiousness and rapid spread of severe acute respiratory syndrome coronavirus. *EID Journal*, 26(7).

Varghese, P. M., Tsolaki, A. G., Yasmin, H., Shastri, A., Ferluga, J., Vatish, M., Madan, T., and Kishore, U. (2020). Host-pathogen interaction in covid-19: Pathogenesis, potential therapeutics and vaccination strategies. *Immunobiology*, page 152008.