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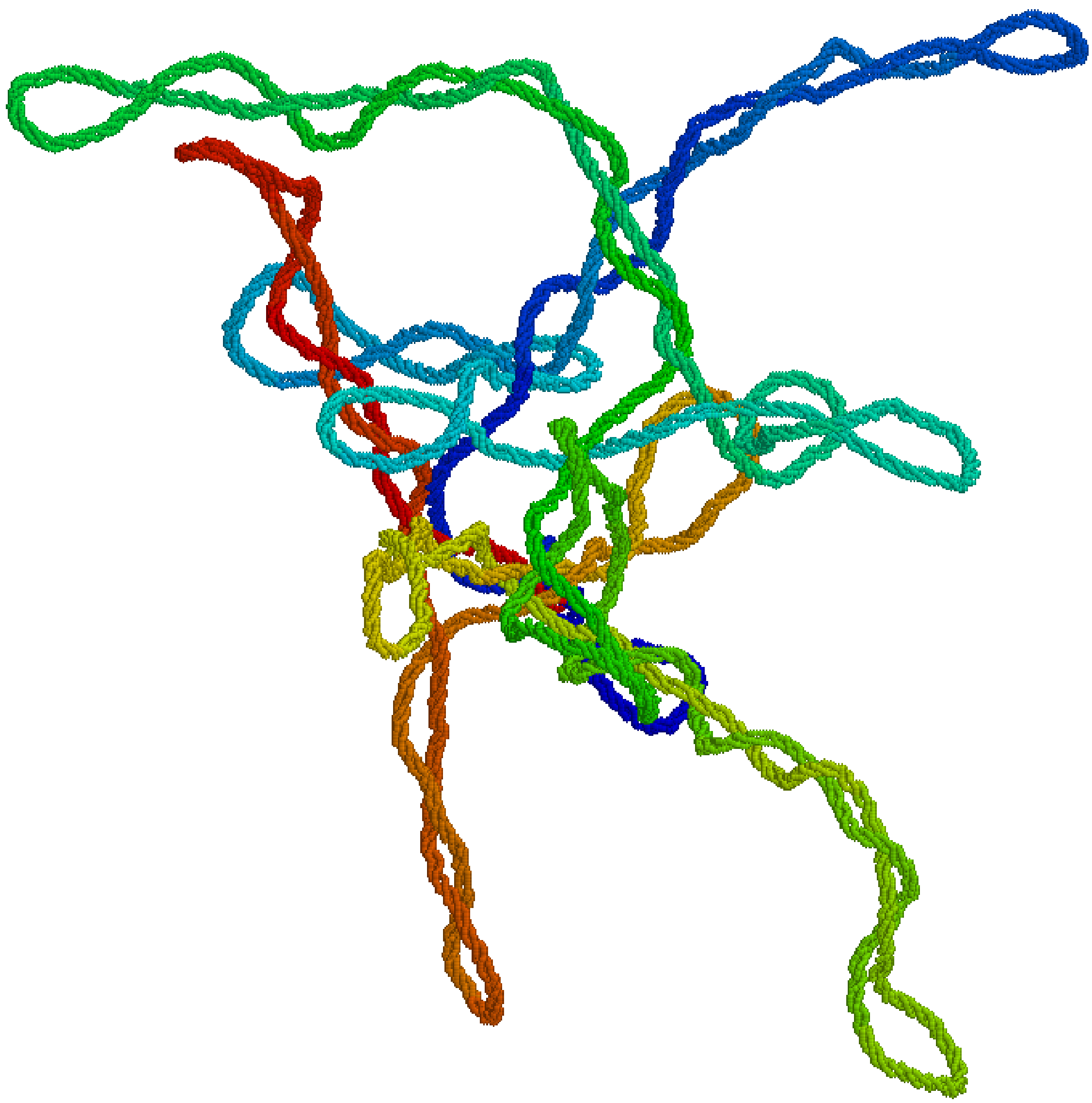

2011

CONFERENCE

2011 MIT-PRIMES CONFERENCE

Program for Research In Mathematics, Engineering, and Science
for High School Students

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9:00 am: Welcoming Remarks

Prof. Michael Sipser, Chair of the MIT Mathematics Department

Prof. Pavel Etingof, PRIMES Chief Research Advisor

Dr. Slava Gerovitch, PRIMES Program Director

9:15 am: Session 1
Dynamic Combinatorics and Geometry

Xiaoyu He, "Rotor-Routers" *(mentor Dr. Tanya Khovanova)*

Ziv Scully, "Progress on the parallel chip-firing problem" *(mentor Yan Zhang)*

Christina Chen, "Hiding behind and hiding inside" *(mentor Dr. Tanya Khovanova)*

10:30 am: Session 2
Combinatorics and Number Theory

Aaron Klein, "Counting matrices with restricted positions by rank over finite fields" *(mentor Alejandro Morales)*

Caroline Ellison, "Polynomial coefficients over finite fields" *(mentor Giorgia Fortuna)*

Saarik Kalia, "Fibonacci numbers and continued fractions" *(mentor Dr. Tanya Khovanova)*

11:45 am: Session 3
Representation Theory I

Sheela Devadas and Carl Lian, "Modular representations of Cherednik algebras associated to symmetric groups" *(mentor Steven Sam)*

Fengning Ding, "Infinitesimal Cherednik algebras" *(mentor Sasha Tsymbaliuk)*

12:30 pm: Lunch break

1:45 pm: Session 4
Representation Theory II: Computer Algebra Lab Projects

Masahiro Namiki, "Determinant formulas for quantum GL(N)" *(mentor David Jordan)*

Surya Bhupatiraju, William Kuszmaul, and Jason Li, "Lower central series of associative algebras in characteristic p" *(mentor David Jordan)*

Michael Zhang and Yongyi Chen, "Poisson homology in characteristic p" *(mentor David Jordan)*

3:00 pm: Session 5
Computational Biology

Dash Elhauge and Andrew Kim, "Modeling the role of cell fusion in cancer development" *(mentor Christopher McFarland)*

Campbell Hewett, Sylvia Hurlimann, Stephanie Palocz, and Dong-Gil Shin, "Complex 3D DNA structures: Computational investigations using Molecular Dynamics (MD) simulations" *(mentors Geoffrey Fudenberg and Maxim Imakaev)*

Saturday, May 21
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2011 PRIMES Conference abstracts

Session 1. Dynamic Combinatorics and Geometry

Xiaoyu He

Rotor-Routers

Mentor Dr. Tanya Khovanova

Project suggested by Prof. James Propp (U. of Mass. at Lowell)

Rotor-routers are a blossoming new field in dynamic combinatorics, giving natural deterministic analogues for stochastic processes such as diffusion-limited aggregation. In 2010 J. Propp defined a concept of universal rotor-router classes. I will introduce these topics and present powerful new tools in the subject. I will discuss the compressor configuration, which has rare period-preserving properties and decides the universality of wide classes of two-state rotor-router types. I will also state the Reduction Theorem that reduces rotors of n states to rotors of two states for the purposes of this problem. Finally, I will state the counter-intuitive conjecture that all unboppy rotor types are universal and present strong empirical evidence for it.

Ziv Scully

Progress on the parallel chip-firing problem

Mentor Yan Zhang

with collaboration of Damien Jiang

Project suggested by Dr. Lionel Levine (MIT)

We consider the ″parallel chip-firing game″: at each vertex of a simple, undirected graph we place a nonnegative integral number of chips. Each turn, every vertex that has at least as many chips as neighbors "fires" one chip to each neighbor. It is clear that every game eventually becomes periodic. In this work we characterize initial positions that lead to periodic games with period 2. Using this we determine the eventual periods of games on trees given only the total number of chips. Finally, we introduce the concept of ″motor vertices″ and study the effects of a particular class of motor vertices connected to a tree.

Christina Chen

Hiding behind and hiding inside

Mentor Dr. Tanya Khovanova

Project suggested by Prof. Daniel Klain (U. of Mass. at Lowell)

A shape A is said to be able to hide behind a shape B if in every direction the shadow of B contains a translate of the corresponding shadow of A. Dan Klain recently published the amazing result that in three dimensions, it is possible for A to be able to hide behind B and have a larger volume than B. Nobody had previously obtained any numerical results, so my project was to find specific examples that would generate the largest volume ratio. The shapes in my examples involved Minkowski sum constructions, and the largest volume ratio I obtained was 1.16.

Session 2. Combinatorics and number theory

Aaron Klein

Counting matrices with restricted positions by rank over finite fields

Mentor Alejandro Morales

Project suggested by Prof. Alexander Postnikov (MIT)

We look at the general problem of finding the number of matrices over a finite field $\mathbf{F}_q$ with certain rank and with support that avoids a set of the entries. These matrices are a q-analogue of permutations with restricted positions (i.e., rook placements) and it is known that for general sets these numbers are not polynomials in q (Stembridge, 1998). However, the numbers are polynomials when the set is a Young diagram (Haglund, 1998), since they are a normalization of rook polynomials: the sum of $q^{\text{inversions}}$ for each rook placement. We extend this result to skew Young diagrams and give conjectures for the cases of the complement of skew Young diagrams and Rothe diagrams of permutations.

Caroline Ellison

Polynomial coefficients over finite fields

Mentor Giorgia Fortuna

Project suggested by Prof. Richard Stanley (MIT)

Coefficients of polynomials are important to study because of their wide-ranging applications in many areas of mathematics, notably combinatorics; for example, the coefficients of the polynomial $(1 + x + x^2)^n$ correspond to the number of ordered trees having $n+1$ leaves, all at level 3 and $n+k+3$ edges. What happens if you look at the coefficients of this polynomial modulo a prime p? In this project, I am investigating the number of nonzero coefficients of this polynomial modulo various primes. I will describe the solution of this problem for $p = 2$ and 3, which was previously known. I also partially succeeded in solving the $p = 5$ case as well as finding solutions for specific values of n and investigating values of p for which $1 + x + x^2$ is reducible (i.e., $p = 3k + 1$).

Saarik Kalia

Fibonacci numbers and continued fractions

Mentor Dr. Tanya Khovanova

Project suggested by Prof. Henry Cohn (MSRNE and MIT)

Even though the Fibonacci numbers F_n are one of the most well known and studied sequences in mathematics, they still have a mysterious property which is not fully understood. Namely, consider the ratio of m-th powers of two consecutive Fibonacci numbers, F_{n+1}^m/F_n^m. As $n \to \infty$, this ratio tends to the m-th power φ^m of the golden ratio φ. Since φ^m is a quadratic irrational, it has a periodic continued fraction expansion, and hence the beginning of the continued fraction expansion of F_{n+1}^m/F_n^m must exhibit this periodic pattern for large n. This, however, says nothing about what happens at the end of the continued fraction expansion of F_{n+1}^m/F_n^m, and at first sight one should expect random behavior there. Surprisingly, this is not the case. For instance, if $m = 1$, the continued fraction expansion of F_{n+1}^m/F_n^m is just the truncation of the expansion of the golden ratio

φ, which consists of ones. For $m > 1$ there is also a clear albeit more complicated pattern, consisting of several alternating short and long strings. In this talk I will describe in detail this peculiar behavior for small values of m (namely, $m = 2, 3, 4, 5$). The description for $m = 4$ and 5 is still conjectural. I will also discuss possible ways of generalizing these results.

Session 3. Representation theory I

Sheela Devadas and Carl Lian

Modular representations of Cherednik algebras associated to symmetric groups

Mentor Steven Sam

Project suggested by Prof. Pavel Etingof (MIT)

We study lowest weight irreducible representations of rational Cherednik algebras attached to the symmetric group S_n in characteristic p, focusing specifically on the case $p \leq n$, which is more complicated than the case $p > n$ (since S_n-modules are not semisimple). The goal of our work is to calculate characters (and in particular Hilbert series) of these representations. By studying the kernel of the contravariant bilinear form on Verma modules, we proved formulas for Hilbert series of irreducible modules in a number of cases, and also obtained a lot of computer data which suggests a number of conjectures. Specifically, we find that the shape and form of the Hilbert series of the irreducible representations and the generators of the kernel tend to be determined by the value of n modulo p. We will present several general conjectures at the end of the talk.

Fengning (David) Ding

Infinitesimal Cherednik algebras

Mentor Sasha Tsymbaliuk

Project suggested by Prof. Pavel Etingof (MIT)

Infinitesimal Cherednik algebras H_c for $\mathfrak{gl}_n$ were introduced by Etingof, Gan and Ginzburg as continuous analogs of the widely studied rational Cherednik algebras. Because H_c is isomorphic to the universal enveloping algebra of the Lie algebra $\mathfrak{sl}_{n+1}$ for certain c, these algebras can be viewed as a deformation of the classical algebra $U(\mathfrak{sl}_{n+1})$. For the case of $\mathfrak{gl}_2$, we present a precise formula for all deformations c so that H_c satisfies the PBW property. We then give an explicit construction of the center of H_c, which Tikaradze proved to be isomorphic to some polynomial algebra. By calculating the action of the center on the Verma module, we compute the determinant of the Shapovalov form to find all irreducible Verma modules. Finally, we classify finite dimensional representations of H_c, and compute their characters.

Session 4. Representation theory II

Masahiro Namiki

Determinant formulas for quantum GL_n

Mentor David Jordan

Project suggested by David Jordan (MIT)

The reflection equation algebra for GL_n is a non-commutative deformation of the coordinate algebra $\mathcal{O}(GL_n)$. This algebra contains an important central element called the quantum determinant, $\det_q$, which in the literature is usually constructed by indirect means. We propose a combinatorial formula for $\det_q$, deforming the usual determinant formula for $\mathcal{O}(GL_n)$, and involving the length and excedance functions on the symmetric group S_n (i.e., the number of inversions of a permutation s, and the number of i such that $s(i) > i$). The formula was discovered by gathering data using the computer algebra system MAGMA, for the coefficients in the range $n = 2, \ldots, 6$, and seeking patterns. The formula has been verified by computer up to $n = 11$, and we are working on its proof in the general case.

Surya Bhupatiraju, William Kuszmaul, and Jason Li

Lower central series of associative algebras in characteristic p

Mentor David Jordan

Project suggested by Prof. Pavel Etingof (MIT)

The lower central series of an associative algebra A is defined inductively by $L_1(A) = A$, $L_i(A) = [A, L_{i-1}(A)]$. This series and its associated graded components $B_i(A) = L_i(A)/L_{i+1}(A)$ were studied recently in the case of the free algebra over complex numbers by Feigin-Shoikhet, Dobrovolska-Kim-Ma and Arbesfeld-Bapat-Jordan. We continue the study of the lower central series and its associated graded components for a free associative algebra with n generators, focusing on algebras defined over fields of characteristic p. New phenomena arise, owing to the existence of p-torsion elements in the integral form of the graded components. We produce several examples of torsion elements, and present a conjecture of P. Etingof, based on our computations, concerning the general form of torsion elements in the second graded component.

Yongyi Chen and Michael Zhang

Poisson homology in characteristic p

Mentor David Jordan

Project suggested by Prof. Pavel Etingof (MIT)

Let G be a finite subgroup of $SL_2(\mathbf{C})$, and p a prime not dividing $|G|$. We compute the Poisson Homology $HP_0 := A/\{A, A\}$ for $A = \overline{\mathbb{F}_p}[x, y]^G$. In particular, we determine an explicit formula for the Hilbert series of HP_0, when $G = C_n$ (the cyclic group of order n) and $G = \mathrm{Dic}_n$ (the dicyclic group of order $4n$). We also state a conjecture describing HP_0 for cones of smooth plane curves of degree d.

Session 5. Computational Biology

Dash Elhauge and Andrew Kim

Modeling the role of cell fusion in cancer development

Mentor Christopher McFarland

Project suggested by Prof. Leonid Mirny (MIT)

Cancer is a disease caused by genetic alterations, or mutations, in normal living cells, which cause cells to divide and grow at an uncontrolled rate. Recent studies have discovered that cell fusion could play a critical role in cancer initiation and progression by allowing mutations which arise in separate cell lineages, to recombine into a new more advanced cancer cell. We developed a mathematical model of cancer progression, incorporating cell fusion, and have computationally investigated how the rate of cell fusion affects initiation and growth of cancer. We found that cell fusion can significantly reduce the waiting time to cancer. Using gene expression profiles from known breast cancers, we tested the role of cell fusion in this disease.

Stephanie Palocz

Segregation of Ring Polymers:
What is the Importance of Entropy in the Bacterial Nucleus

Mentors Geoffrey Fudenberg and Maxim Imakaev

Project suggested by Prof. Leonid Mirny (MIT)

Previous work by Jun et al. proposed an entropic driving force for the segregation of E. coli's circular chromosomes during cell division. Using molecular dynamics simulations in pyOpenMM on a graphics processing unit (GPU), we leverage vastly increased computational power to improve understanding of chromosome segregation in bacteria. First, we developed an algorithm to model the process of chromosome replication by growing two un-entangled DNA rings. We then used molecular dynamics to study the segregation of these rings into two daughter cells. By varying length and stiffness of the polymer, and the geometry of the confining capsid (i.e. the shape of a bacterial nucleoid), we comprehensively investigate the conditions where two confined ring polymers will separate. Our results will have important implications for understanding chromosome segregation in bacteria.

Sylvia Hürlimann

The Structure of DNA in E. Coli

Mentors Geoffrey Fudenberg and Maxim Imakaev

Project suggested by Prof. Leonid Mirny (MIT)

Although the E. coli genome has been extensively studied, we are only now beginning to understand its three-dimensional organization. Prokaryotic chromosomes lack the histones and nuclear envelope that help organize the DNA in eukaryotic cells, yet experimental data shows that the chromosome of a prokaryotic cell is organized linearly within the nucleoid. The recently proposed Fluctuating Filament Model (Wiggins et al. 2010) attributes the observed linear DNA organization to the confinement of the chromosome within the nucleoid and the tethering of the DNA at either end of the cell. Using molecular dynamics simulations in pyOpenMM, a more powerful method than those previously available, we studied a model E. coli genome consisting of a polymer that was confined inside a cylinder and had its ends tethered. Our simulations show that tethering of the ends and confinement of the polymer caused the polymer to be arranged linearly along the axis of the cylinder, supporting the possibility of the Fluctuating Filament Model and giving a viable theoretical model for DNA organization in prokaryotic cells.

Dong-Gil Shin

Scaffold Assisted Chromosome Condensation: Molecular Dynamics Simulations

Mentors Geoffrey Fudenberg and Maxim Imakaev

Project suggested by Prof. Leonid Mirny (MIT)

Despite decades of experimentation, the mechanism by which chromatin condenses and separates into two sister chromatids during cell division remains undescribed. Axial protein scaffolds have long been suspected to play a role in this process. In order to investigate the feasibility of chemically identical scaffolds separating identical sister chromatids, we studied a model where two linear scaffolds placed in parallel exerted a short-range attractive force on two polymer chains. Molecular dynamics simulations were performed using pyOpenMM to make use of powerful GPU capabilities. Our simulations show that successful and rapid separation of chromosomes is possible, which suggests a novel mechanism of chromosomes assembly during cell division.

Campbell Hewett

Simulating Supercoiling in Prokaryotic DNA

Mentors Geoffrey Fudenberg and Maxim Imakaev

Project suggested by Prof. Leonid Mirny (MIT)

Supercoiling has several important roles in a prokaryotic cell, including DNA packing and DNA replication, and occurs naturally in response to enzymatically induced twisting. We extend the current understanding of supercoiling in bacterial chromosomes by developing a realistic coarse-grained model for the structure of a DNA polymer. The model allows to obtain and study conformations of supercoiled DNA. Extensive tests of the model showed that the polymer maintains the same total twist throughout the simulation, meaning that there is no release of torsion through inaccuracies in the program. Our model of supercoiling will allow us to test the biologically important proposal that supercoiling can help us drive chromosome segregation in E. coli during DNA replication.

The front page: A supercoiled pleconpremic DNA from molecular dynamics simulations. Twisted DNA naturally relaxes into this highly entwined state, similar to filaments often formed by garden hoses or telephone cords. Image from a PRIMES project by Campbell Hewett.