

MIT PRIMES STEP Junior Group

The Struggles Of Chessland

PRIMES Conference, May 22nd, 2022



Irene Choi, Shreyas Ekanathan, Aidan Gao, Sylvia Zia Lee, Rajarshi Mandal, Vaibhav Rastogi,

Daniel Sheffield, Michael Yang, Angela Zhao, Corey Zhao

Mentored by Dr. Tanya Khovanova



Bermuda

Apparent
location of the
Bermuda
Triangle

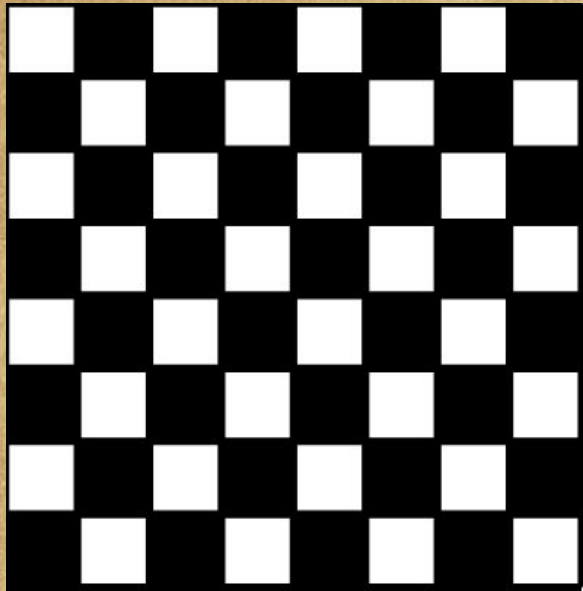
Miami

**San Juan,
PUERTO RICO**

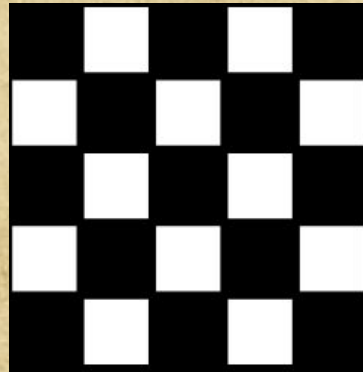
CUBA



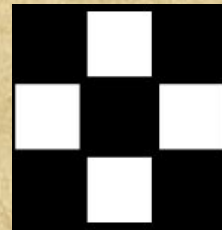
MIT PRIMES STEP *Junior Group: The Struggles of Chessland*



Island 8



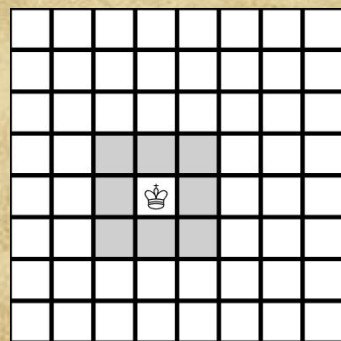
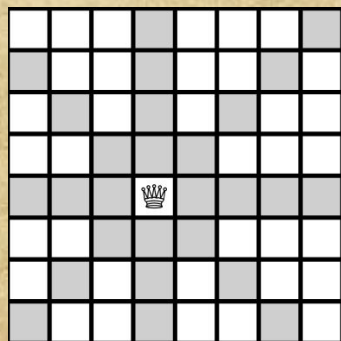
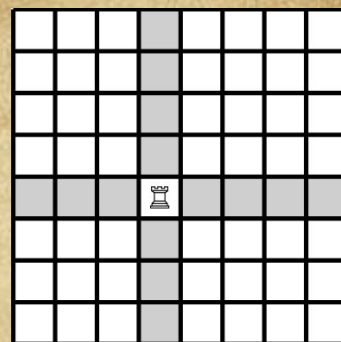
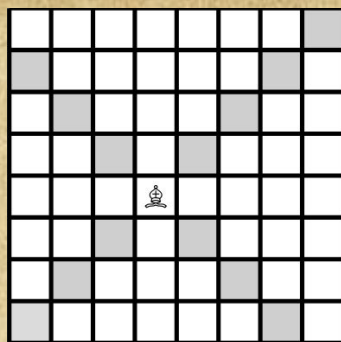
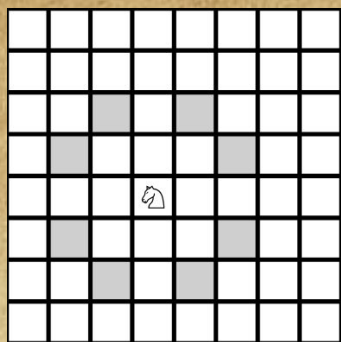
Island 5



Island 3



Island 2



Surveying

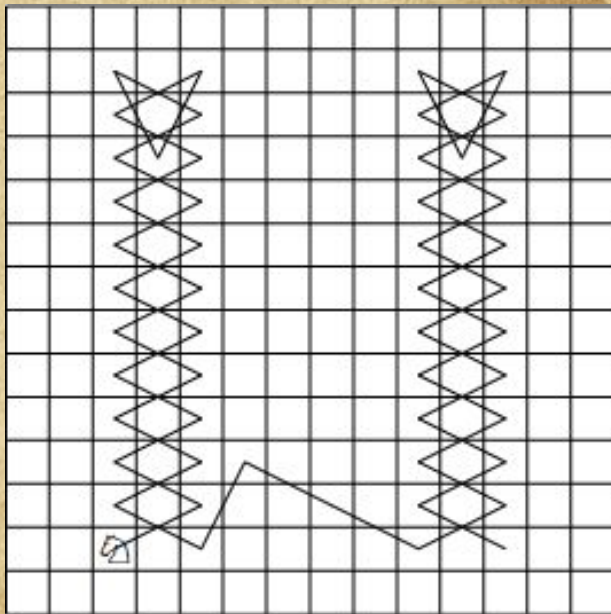


MIT PRIMES STEP *Junior Group: The Struggles of Chessland*

Shoelace Formula (Pt 1)

Shoelace in $7 \times k$ rectangle: $2k - 3$

Islands of size $7k$: $14k^2 - k - 2$



Shoelace Formula (Pt 2)

Islands of size $7k - 1$: $14k^2 - 3k - 3$

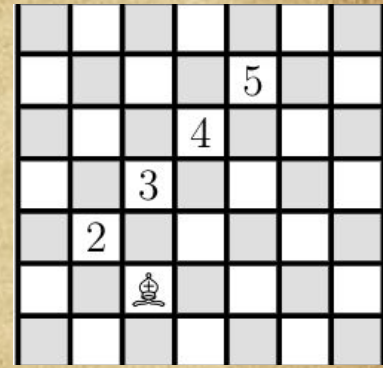
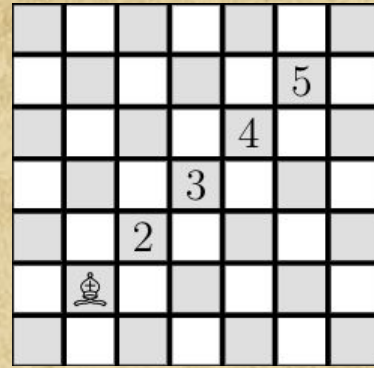
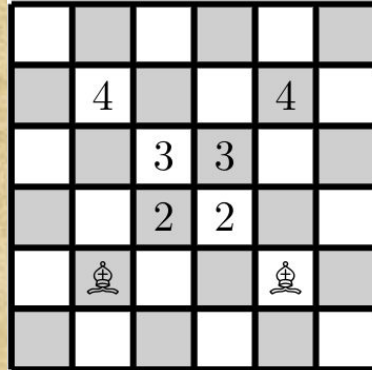
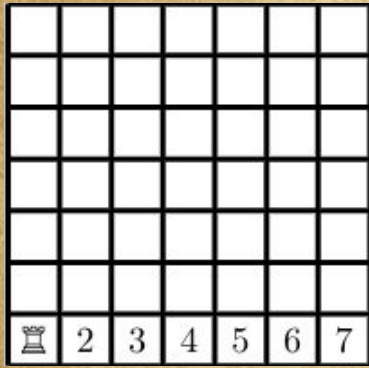
Islands of size $7k - r$: $14k^2 - (2r + 1)k - 2$

Islands of size $7k - 6$: $14k^2 - 20k + 4$

Note: When squares are repeated, the larger number is put in the square (61, 65, and 68 here.)

									70	
	13	15				41	44	67		
	16	12				45	41		69	
	11	14	17			40	43	68		66
	18	10				47	65			
	9	19				38	48		64	
	20	8				49	37			
	7	21				36	50	63		
	22	6				51	35			
	5	23				34	52		62	
	24	4				53	61			
	3	25	28			32	54	57	60	
	26	2			29	55	31			58
	2	27				30	56	59		

The Rook and the Bishop



The King: Part 1

Straight Spiral

(Divide n by 6, and let k be the quotient and r be the remainder)

Let $f(n)$ denote the number of moves to survey Island n

$$n = 6k + r$$

Recursive Formula: $f(n+6) = f(n) + 4(n+3)$

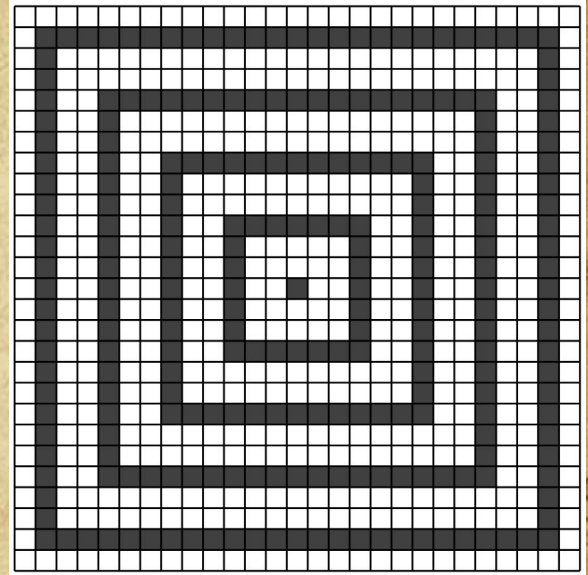
General Formula:

$$f(n) = 16k^2 + 4rk + s(r)$$

$$s(0) = -2 \qquad s(3) = 1$$

$$s(1) = -1 \qquad s(4) = 4$$

$$s(2) = 0 \qquad s(5) = 7$$



The King: Part 2

Zig-zag Spiral

(Divide n by 8, and let K be the quotient and R be the remainder)

Let $g(n)$ denote the number of moves to survey Island n

$$n = 8K + R$$

Recursive Formula: $g(n+8) = g(n) + 4(n+5)$

General Formula:

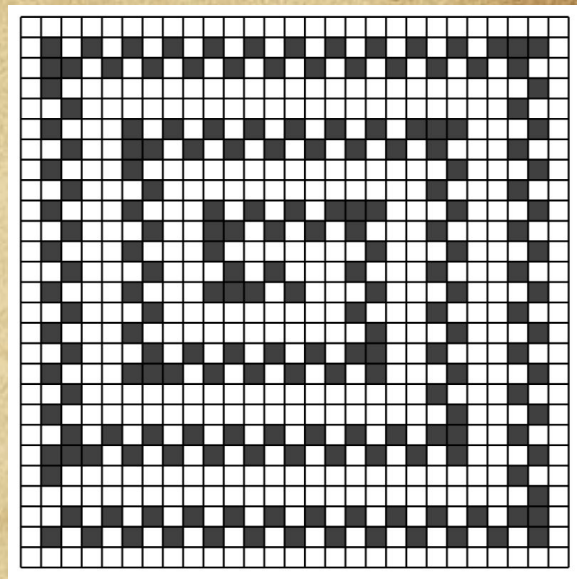
$$g(n) = 16K^2 + (4R + 4)K + t(R)$$

$$t(0) = -2 \quad t(4) = 4$$

$$t(1) = -1 \quad t(5) = 8$$

$$t(2) = 1 \quad t(6) = 11$$

$$t(3) = 2 \quad t(7) = 14$$

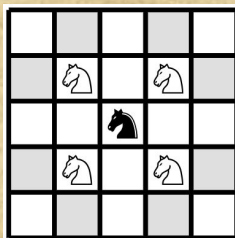
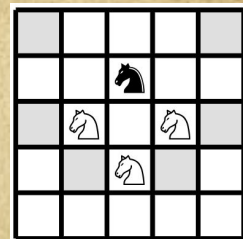
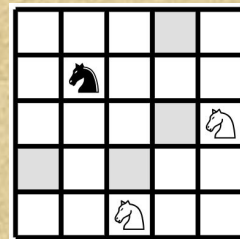
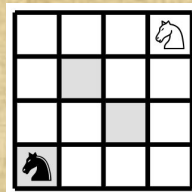
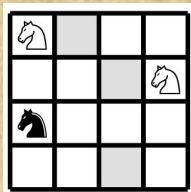




MIT PRIMES STEP *Junior Group: The Struggles of Chessland*

Knights

1	2	2	2	...	2	2	2	1
2	2	3	3	...	3	3	2	2
2	3	4	4	...	4	4	3	2
2	3	4	4	...	4	4	3	2
:	:	:	:	...	:	:	:	:
2	3	4	4	...	4	4	3	2
2	3	4	4	...	4	4	3	2
2	2	3	3	...	3	3	2	2
1	2	2	2	...	2	2	2	1

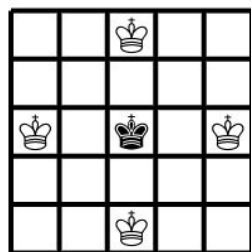
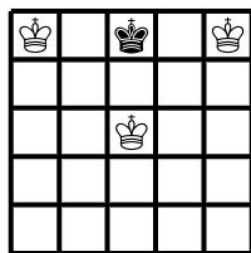
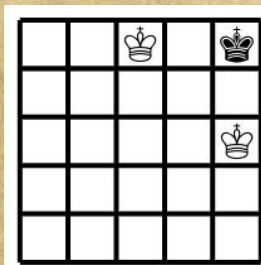
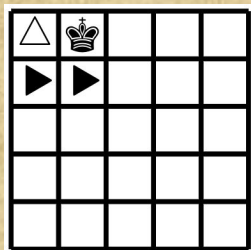
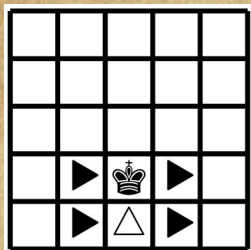


Bishops

0	n-2	n-3	n-4	n-5	...	n-5	n-4	n-3	n-2	0
n-2	0	0	n-3	n-4	...	n-4	n-3	0	0	n-2
n-3	0	0	0	0	0	0	n-3
n-4	n-3	0	0	0	n-3	0	0	0	n-3	n-4
n-5	n-4	...	0	0	0	0	0	...	n-4	n-5
:	:	:	n-3	0	0	0	n-3	:	:	:
n-5	n-4	...	0	0	0	0	0	...	n-4	n-5
n-4	n-3	0	0	0	n-3	0	0	0	n-3	n-4
n-3	0	0	0	0	0	0	n-3
n-2	0	0	n-3	n-4	...	n-4	n-3	0	0	n-2
0	n-2	n-3	n-4	n-5	...	n-5	n-4	n-3	n-2	0

0	n-2	n-3	n-4	n-5	n-5	n-4	n-3	n-2	0
n-2	0	0	n-3	n-4	n-4	n-3	0	0	n-2
n-3	0	0	0	n-3	n-3	0	0	0	n-3
n-4	n-3	0	0	0	n-3	n-3	0	0	0	n-3	n-4
n-5	n-4	n-3	0	0	0	0	0	0	n-3	n-4	n-5
:	n-3	0	0	0	0	n-3	:
:	n-3	0	0	0	0	n-3	:
n-5	n-4	n-3	0	0	0	0	0	0	n-3	n-4	n-5
n-4	n-3	0	0	0	n-3	n-3	0	0	0	n-3	n-4
n-3	0	0	0	n-3	n-3	0	0	0	n-3
n-2	0	0	n-3	n-4	n-4	n-3	0	0	n-2
0	n-2	n-3	n-4	n-5	n-5	n-4	n-3	n-2	0

Kings



2	0	3	3	...	3	3	0	2
0	0	0	0	...	0	0	0	0
3	0	4	4	...	4	4	0	3
3	0	4	4	...	4	4	0	3
:	:	:	:	...	:	:	:	:
3	0	4	4	...	4	4	0	3
3	0	4	4	...	4	4	0	3
0	0	0	0	...	0	0	0	0
2	0	3	3	...	3	3	0	2

The Queen's to-do list:

- Eat (2 hrs)
- Beauty sleep (1 hr)
- Get makeup applied (8 hrs)
- Beauty sleep (1 hr)
- Pick out clothes (8 hrs)
- Sleep (4 hrs)



Queens

2	2	2	2
2	0	0	2
2	0	0	2
2	2	2	2

3	2	2	2	3
2	2	3	2	2
2	3	4	3	2
2	2	3	2	2
3	2	2	2	3

3	3	3	3	3	3
3	3	3	3	3	3
3	3	4	4	3	3
3	3	4	4	3	3
3	3	3	3	3	3
3	3	3	3	3	3

Acknowledgements

Thank you to the MIT PRIMES STEP program and Dr. Tanya Khovanova for providing us with this opportunity.

Special Thanks to:

Our Family and Friends,
Especially our Parents.



Thank You!

Any Questions?

