Triangles in Various Geometries

Ben Stokes and Sebastian Cuervo
Primes Circle
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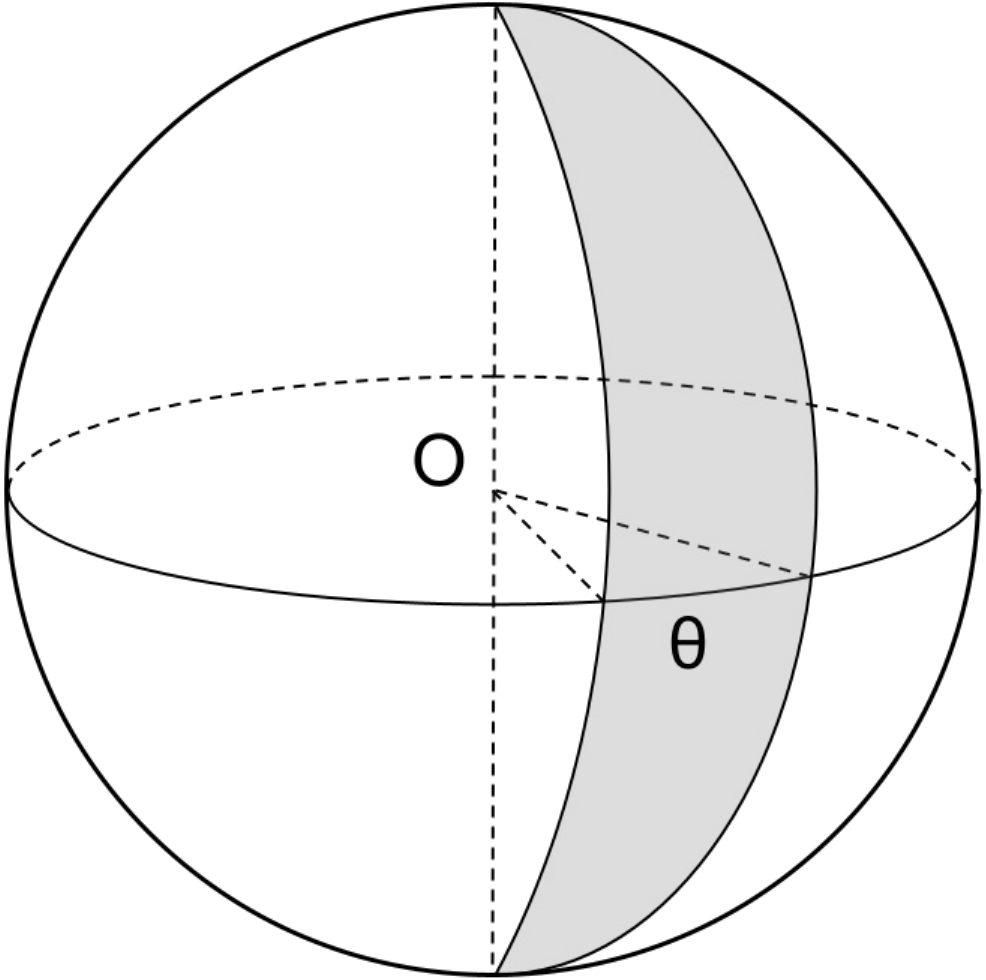
Spherical Geometry

Spheres and Great Circles

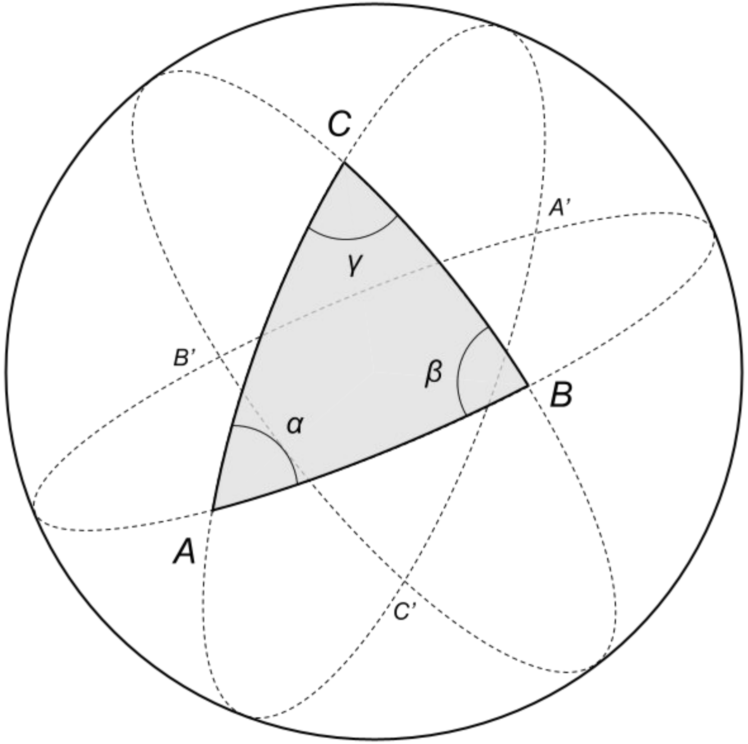
-Great Circles

-Antipodal Points

Lunes



Triangle in Spherical Geometry (Area)



Area of a Spherical Triangle

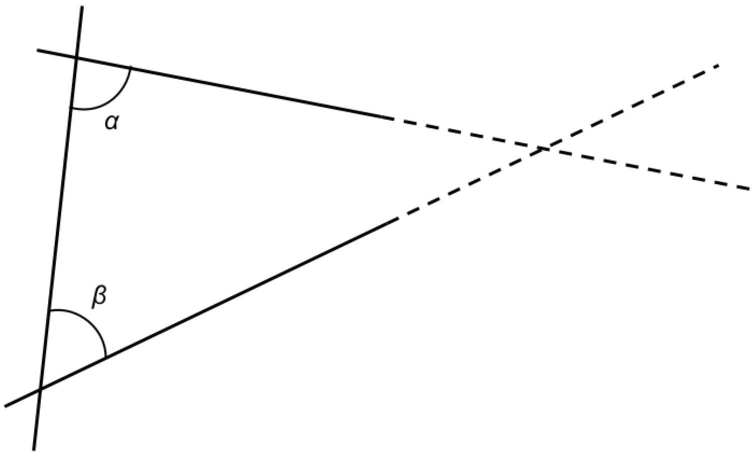
$$
\begin{aligned}
2\pi r^2 &= Area(ABA'C) + Area(BAB'C) + Area(CAC'B) - 2Area(\triangle ABC) \\
2\pi r^2 &= 2\alpha r^2 + 2\beta r^2 + 2\gamma r^2 - 2Area(\triangle ABC) \\
2Area(\triangle ABC) &= 2\alpha r^2 + 2\beta r^2 + 2\gamma r^2 - 2\pi r^2 \\
Area(\triangle ABC) &= (\alpha + \beta + \gamma - \pi)r^2.
\end{aligned}
$$

Axiomatic Systems

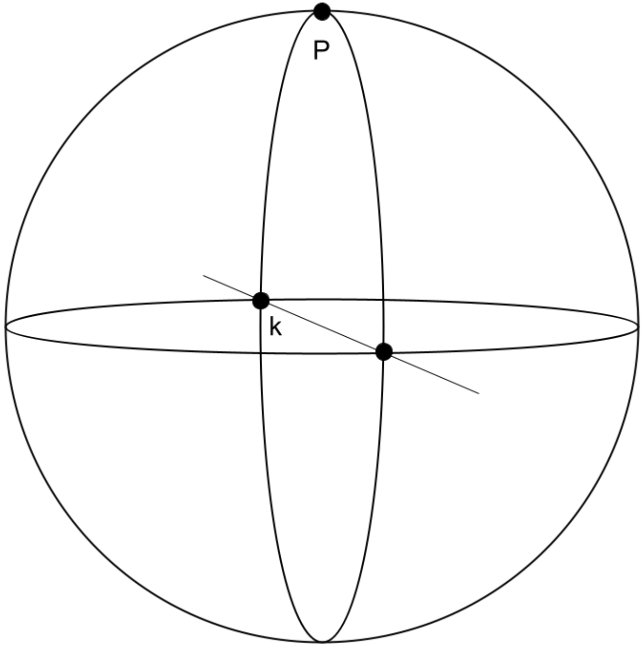
Euclid’s Postulates

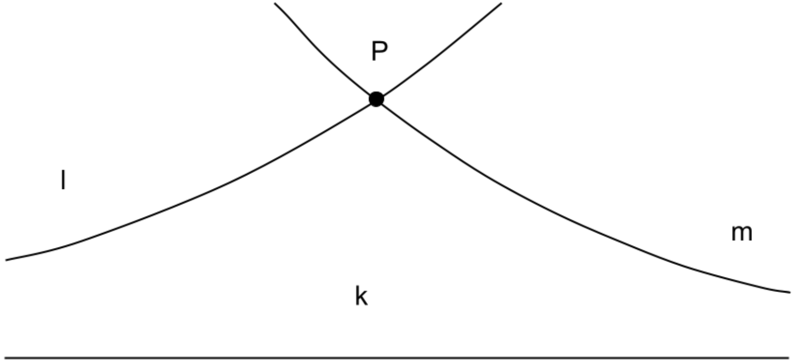
1. We can draw a straight line from any point to any other point.
2. We can continue a line segment continuously into a straight line.
3. We can construct a circle so that every point along the edge is equidistant from the center.
4. The measure of right angles are always equal.
5. **If we have a line intersecting two other lines at two distinct points, and the sum of the measure of the interior angles formed between them is less than π, then the lines will intersect on that side (depicted in next slide).**

The Fifth Postulate



Saccheri's Conclusions

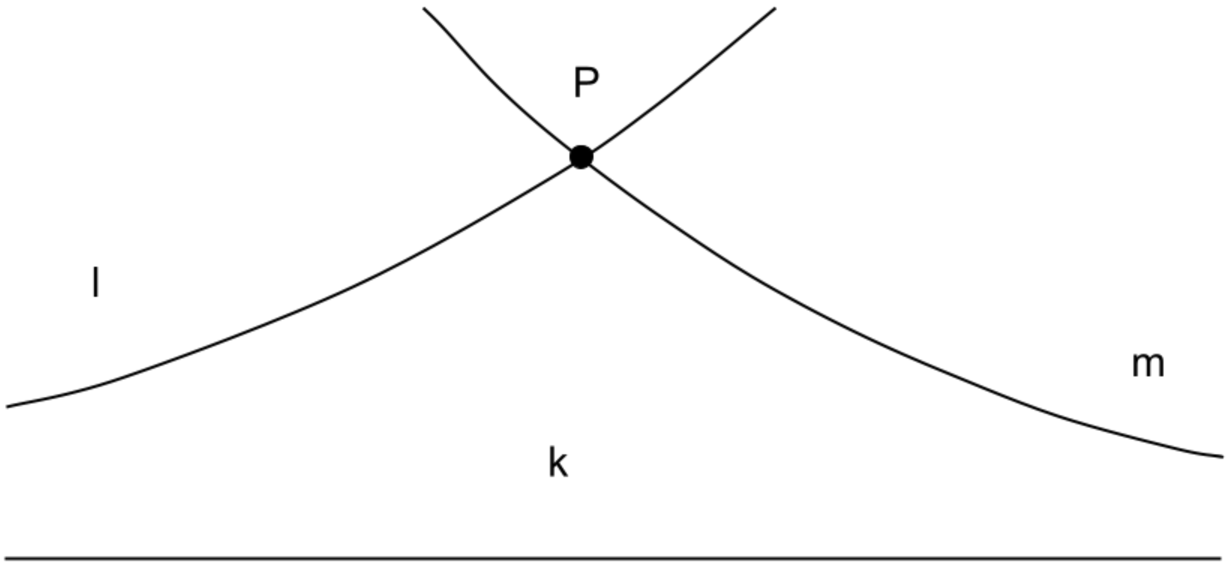




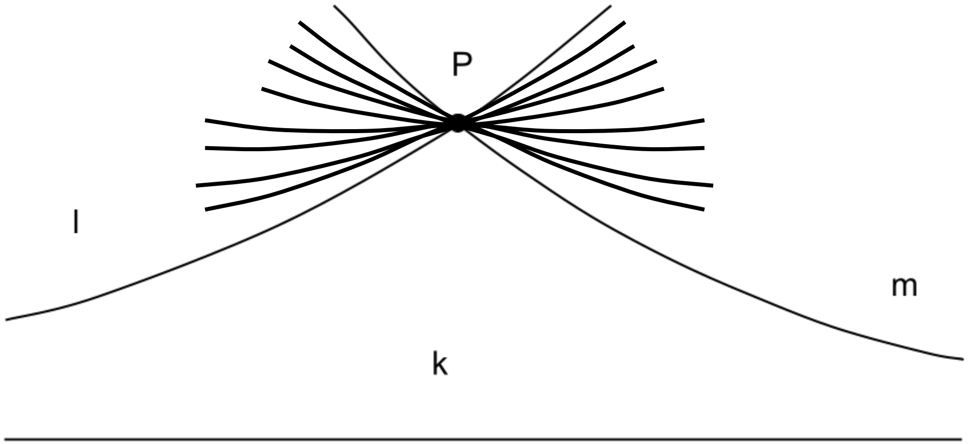
Hyperbolic Geometry

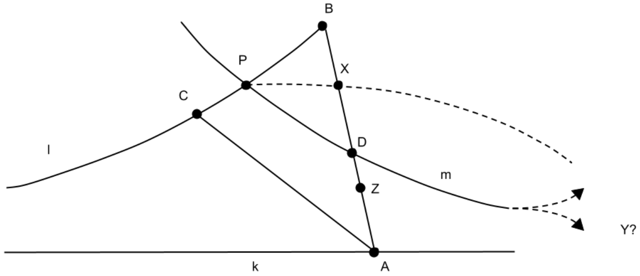
The Characteristic Axiom of Hyperbolic Geometry

Given a line *k* and a point *P* not on *k*, there exists at least two lines *m* and *l* that do not intersect *k*.



Infinitely Many Lines

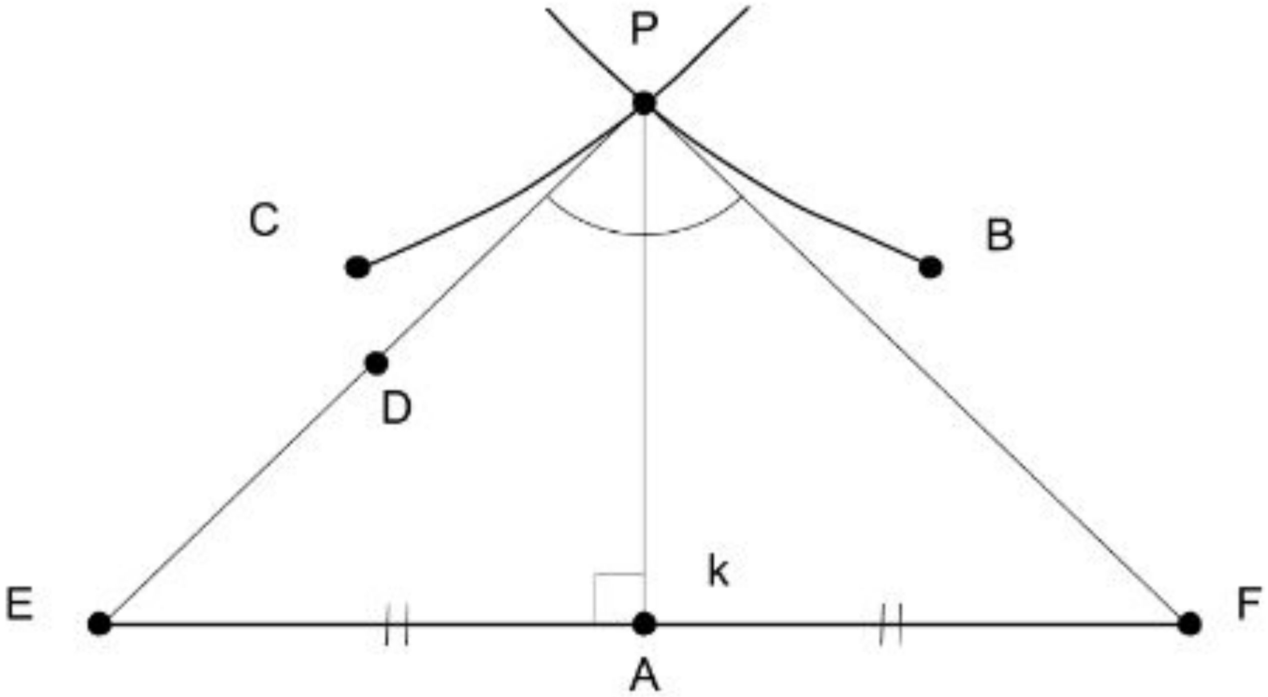




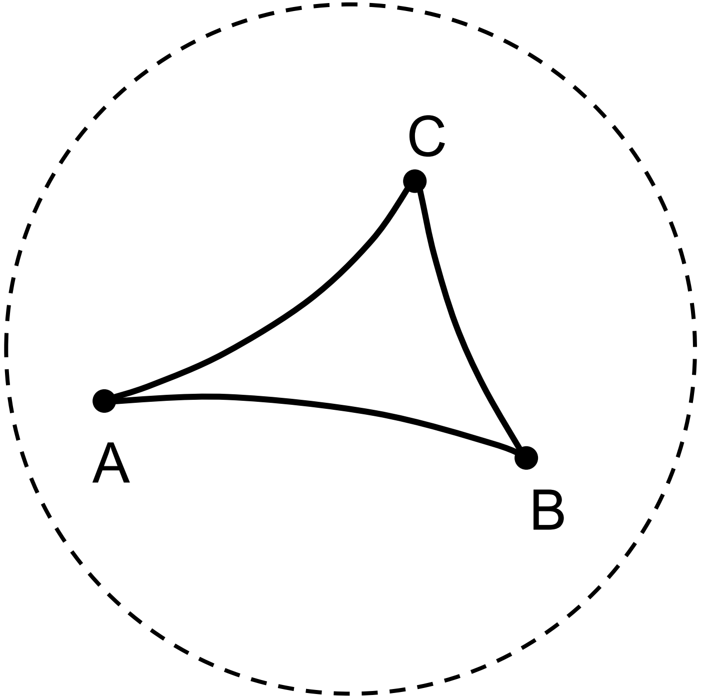
Sensed Parallels

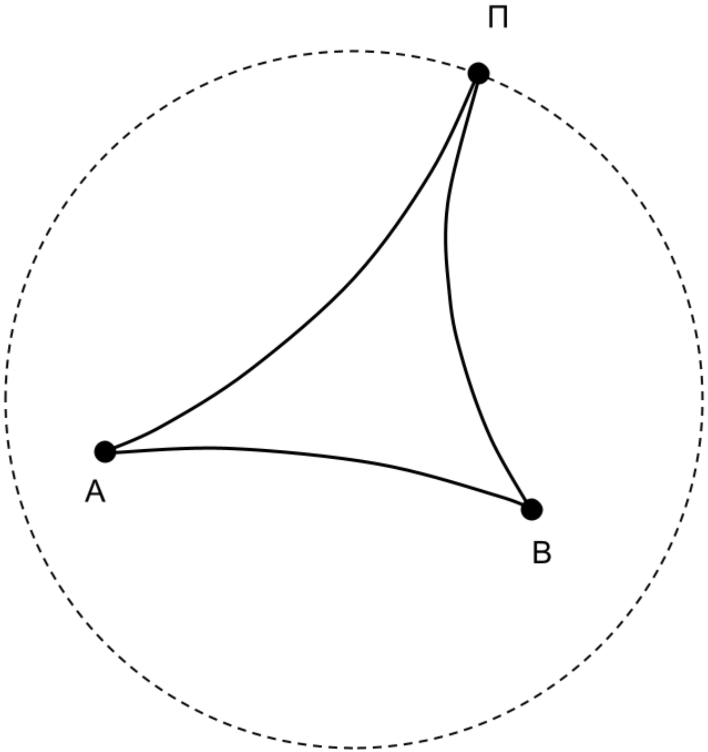
Since there are infinitely many parallel lines in hyperbolic space, how close can a line get before intersecting?

Angles of Parallelism

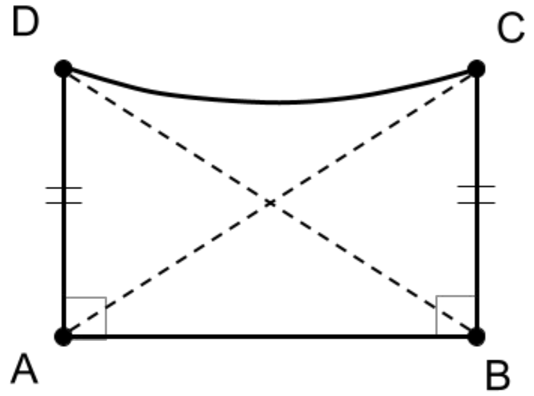


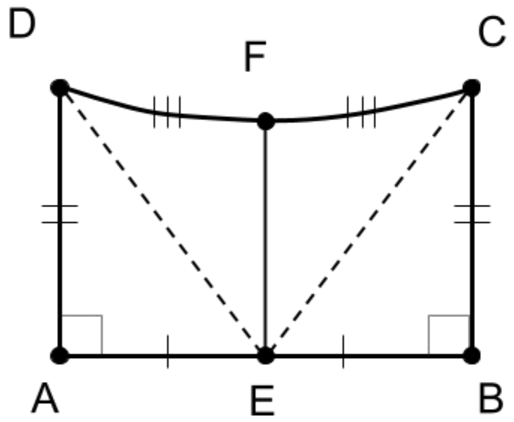
Triangle in Poincaré Model



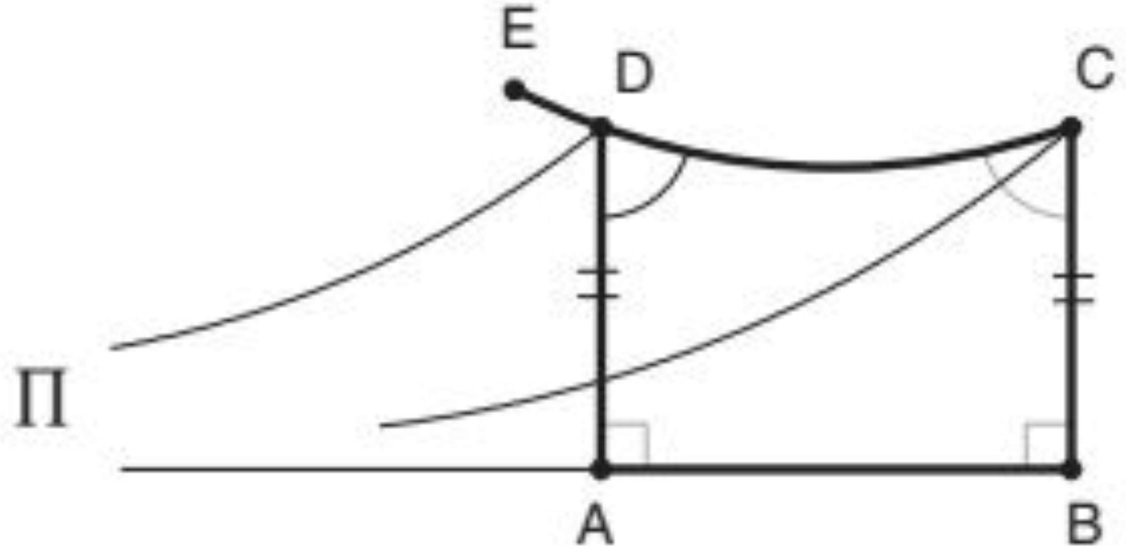


Saccheri Quadrilaterals

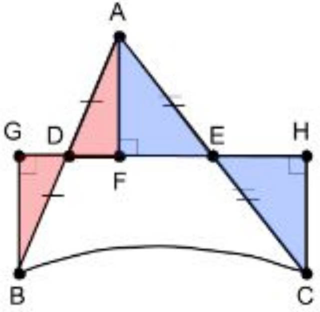


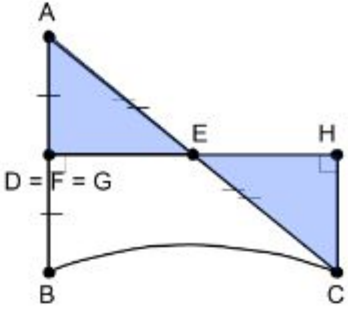


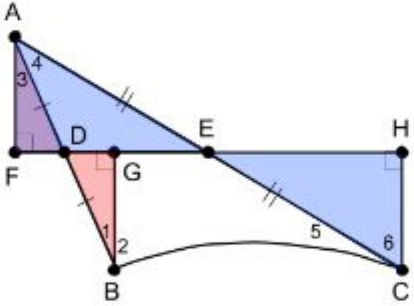
Summit Angles



Triangle Sum Less Than Pi







Thank you!

Any Questions?